MAP 4170 Name:________________________
Test 1 Date: January 28, 2020

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Two 180-day Treasury Bills, one Canadian and the other US, have the same price and the same redemption value. The quoted rate on the US bill is 8%. Determine the quoted rate on the Canadian bill. $= q$

(A) 8.25%

(B) 8.35%

(C) 8.45%

(D) 8.55%

(E) 8.65%

Can: $P\left(1+q\cdot\frac{180}{365}\right)=C$

U.S. $P = C\cdot\left(1-.08\cdot\frac{180}{360}\right) = .96C$

$\therefore\ .96\cancel{C}\cdot\left(1+q\cdot\frac{180}{365}\right)=\cancel{C}$

$q = .08449\ldots$

C

2. An account credits interest using $\delta_t = \frac{0.03}{1+0.03t}$. Mark deposits 1000 at time $t = 0$ and another 2000 at time $t = 3$. Determine the amount of interest Mark earns from time $t = 1$ to time $t = 5$.

(A) 230

(B) 260

(C) 680

(D) 1230

(E) 1260

$\delta_t = \frac{.03}{1+.03t} \Rightarrow a(t) = 1+.03t$

Let A_n = amount Mark has at time n

$A_1 = 1000\cdot a(1) = 1000(1.03) = 1030$

$A_5 = 1000\cdot a(5) + 2000\cdot\frac{a(5)}{a(3)}$

$= 1000\cdot(1.15) + 2000\cdot\frac{1.15}{1.09} = 3260$

Since Mark deposited 2000 at $t=3$, then

$I_{[1,5]} = (3260-1030) - 2000 = 230$

A

3. The annual force of interest credited to a savings account is defined by $\delta_t = 0.026t$ with t in years. A deposit of X into this account at time $t = 0$ doubles to $2X$ at time $t = n$. Determine n.

(A) 6.7

(B) 6.9

(C) 7.1

(D) 7.3

(E) 7.5

$X \cdot a(n) = 2X \Rightarrow a(n) = 2$

$\therefore e^{\int_0^n .026t\,dt} = 2$

$\Rightarrow e^{.013 \cdot t^2 |_0^n} = 2$

$\Rightarrow e^{.013n^2} = 2$

$\Rightarrow n = 7.3$ D

4. Let S be the accumulated value of 1000 invested for one year at a nominal annual rate of discount d convertible quarterly, which is equivalent to an annual effective interest rate of i. Let T be the accumulated value of 1000 invested for two years at a nominal annual rate of discount d convertible semiannually. Given $\frac{S}{T} = \left(\frac{18}{19}\right)^4$, determine i.

(A) 4.1%

(B) 8.2%

(C) 11.4%

(D) 16.6%

(E) 22.8%

$S = 1000\left(1 - \frac{d}{4}\right)^{-4}$

$T = 1000\left(1 - \frac{d}{2}\right)^{-4}$

$\therefore \frac{S}{T} = \left(\frac{1-\frac{d}{4}}{1-\frac{d}{2}}\right)^{-4} = \left(\frac{1-\frac{d}{2}}{1-\frac{d}{4}}\right)^4 = \left(\frac{18}{19}\right)^4$

$\Rightarrow 19 \cdot \left(1 - \frac{d}{2}\right) = 18 \cdot \left(1 - \frac{d}{4}\right) \Rightarrow d = 0.2$

$i =$ aeir and $d^{(4)} = 0.2$

$\Rightarrow (1+i) =$ aaf $= \left(1 - \frac{0.2}{4}\right)^{-4}$

$\Rightarrow i = 0.2277\ldots$ E

5. Account A credits interest using a simple interest rate of i for the first half of year 1, and a nominal interest rate of i compounded semiannually thereafter. Account B credits interest using a nominal interest rate of i compounded annually. A deposit of 500 is made into Account A and a separate deposit of 500 is made into Account B, each at the beginning of year 1. Account A has a value of 820 after 10 years. Determine the amount in Account B after 10 years.

(A) 810

(B) 815

(C) 820

(D) 825

(E) 830

A: $820 = 500\left(1+i\cdot\frac{1}{2}\right)\left(1+\frac{i}{2}\right)^{19}$

$\Rightarrow 820 = 500\left(1+\frac{i}{2}\right)^{20} \Rightarrow i = .05$

B: $i = .05$ aeir

$A(10) = 500(1.05)^{10} = 815$

B

6. The terms of a settlement require that an amount A is to be paid in one year and an additional amount B is to be paid one year after the payment of A is made (i.e. in two years). Using an annual effective discount rate $d = 0.05$, the (total) present value of the payments is 6460, whereas using an annual effective discount rate $d = 0.10$, the (total) present value of the payments is 5940. Determine A.

(A) 3000

(B) 3250

(C) 3500

(D) 3750

(E) 4000

$\begin{cases} \text{aedr} = d = .05 \Rightarrow \text{adf} = v = .95 \\ 6460 = Av + Bv^2 \Rightarrow 6460 = .95A + .9025B \end{cases}$

$\begin{cases} \text{aedr} = d = .1 \Rightarrow \text{adf} = v = .9 \\ 5940 = Av + Bv^2 \Rightarrow 5940 = .9A + .81B \end{cases}$

$(6460 = .95A + .9025B)\cdot(.81)$

$-\ (5940 = .9A + .81B)\cdot(.9025)$

$-128.25 = -.04275A$

$\Rightarrow A = 3000$

A

7. An account credits interest using a simple discount rate of d, where $0 < d < 0.10$. You are given that $i_2 + d_1 = 0.111$ where i_2 is the annual effective interest rate for the second year, and d_1 is the annual effective discount rate for the first year. If 1000 is invested into the account at time 0, determine the accumulated amount in the account after one year.

$$a(t) = (1-dt)^{-1}$$

(A) 1050.0

$$A(1) = 1000(1-d)^{-1}$$

(B) 1052.5

(C) 1055.0

$$i_2 = \frac{a(2)-a(1)}{a(1)} \quad d_1 = \frac{a(1)-\cancel{a(0)}^{=1}}{a(1)} \Rightarrow i_2 + d_1 = \frac{a(2)-1}{a(1)}$$

(D) 1057.5

(E) 1060.0

$$\frac{a(2)-1}{a(1)} = \frac{(1-2d)^{-1}-1}{(1-d)^{-1}} = \frac{\frac{1}{1-2d}-1}{\frac{1}{1-d}} = \frac{\frac{2d}{1-2d}}{\frac{1}{1-d}} = \frac{2d(1-d)}{1-2d}$$

$$\therefore \frac{2d(1-d)}{1-2d} = .111 \Rightarrow 2d - 2d^2 = .111 - .222d$$

$$\Rightarrow 2d^2 - 2.222d + .111 = 0 \Rightarrow d = .052429\ldots$$

$$\therefore A(1) = 1000(1-d)^{-1} = 1055$$

(C)

8. Determine the nominal interest rate compounded quarterly that is equivalent to $d^{(2)}$.

$i^{(4)}$

(A) $i^{(4)} = 4\left[\left(1 - \frac{d^{(2)}}{2}\right)^{\frac{-1}{4}} - 1\right]$

(B) $i^{(4)} = 4\left[\left(1 - \frac{d^{(2)}}{2}\right)^{\frac{-1}{2}} - 1\right]$

$$qaf = 1 + \frac{i^{(4)}}{4} = \left(1 - \frac{d^{(2)}}{2}\right)^{-1/2}$$

(C) $i^{(4)} = 2\left[\left(1 - \frac{d^{(2)}}{2}\right)^{\frac{-1}{4}} - 1\right]$

$$\Rightarrow i^{(4)} = 4 \cdot \left[\left(1 - \frac{d^{(2)}}{2}\right)^{-1/2} - 1\right]$$

(D) $i^{(4)} = 2\left[\left(1 - \frac{d^{(2)}}{2}\right)^{\frac{-1}{2}} - 1\right]$

(B)

(E) None of the above

9. Account A credits interest using a simple interest rate equal to 9%. Account B credits interest using a nominal interest rate of 9%, compounded monthly. Determine the time at which the forces of interest in the two accounts are equal.

(A) 0.01

(B) 0.02

(C) 0.03

(D) 0.04

(E) 0.05

$$\delta_t^A = \frac{.09}{1+.09t}$$

$$\delta_t^B = \ln(aaf) = \ln\left(\left(1+\frac{.09}{12}\right)^{12}\right) = 12\cdot\ln(1.0075)$$

$$\therefore \frac{.09}{1+.09t} = 12\cdot\ln(1.0075)$$

$$\Longrightarrow t = .0416\ldots$$

D

10. Steve makes the following transactions into an account that credits interest using a quarterly effective interest rate of 2%:

Date	Transaction
01/01/2019	Initial Deposit of 1000
07/01/2019	Withdrawal of 400
10/01/2019	Deposit of X

$qaf = 1.02$

As of 01/01/2020, Steve had 1,175 in the account. Determine X.

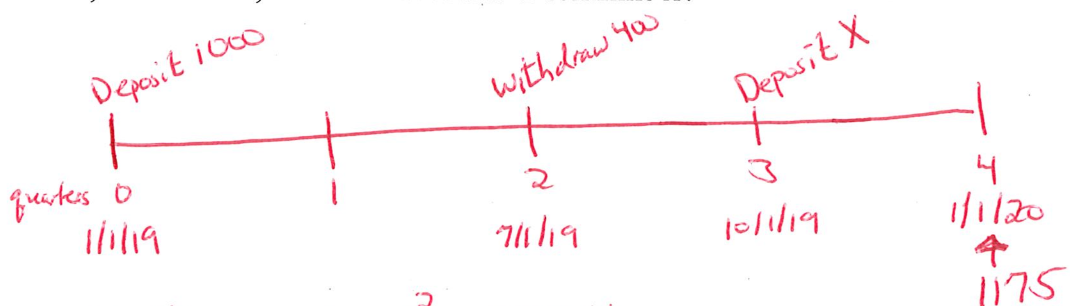
(A) 460

(B) 470

(C) 480

(D) 490

(E) 500



$$A(2^-) = 1000(1.02)^2 = 1040.4$$

$$A(2^+) = 1040.4 - 400 = 640.4$$

$$A(3^-) = 640.4(1.02) = 653.208$$

$$A(3^+) = 653.208 + X$$

$$A(4) = (653.208 + X)(1.02) = 1175$$

$$\Longrightarrow X = 498.75$$

E