MAP 4170 Name: KEY

Test 1 Date: January 28, 2021

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Jack is to receive payments of 5000 in 5 years and 10,000 in 10 years. Using an interest rate of 8% compounded quarterly, determine the present value of the payments.

   (A) 7,755

   (B) 7,825

   (C) 7,895

   (D) 7,965

   (E) 8,035

5000 10000

yrs 0 5 10

quarters 0 20 40

$i^{(4)} = .08$

$\Rightarrow$ qeir $= .02$

$PV = 5000(1.02)^{-20} + 10000(1.02)^{-40}$

$= 7893.76$

2. An account credits interest using $\delta_t = \frac{0.05}{1+0.05t}$ where t is the number of years after January 1, 2020. If 100 is deposited into the account on July 1, 2020, determine the accumulated value of the deposit on January 1, 2025.

   (A) 121.95

   (B) 122.15

   (C) 122.50

   (D) 123.05

   (E) 123.45

$\delta_t = \frac{.05}{1+.05t} \Rightarrow a(t) = 1 + .05t$

1/1/2020 ~ $t = 0$

7/1/2020 ~ $t = \frac{1}{2}$

1/1/2025 ~ $t = 5$

$AV = 100 \cdot paf_{1/2}^{5} = 100 \frac{a(5)}{a(1/2)} = 100 \cdot \frac{1+.05(5)}{1+.05(1/2)}$

$\therefore AV = 121.95$

3. The quoted rate for a 180-day Canadian T-Bill is twice the quoted rate for a 180-day U.S. T-Bill. A 180-day U.S. T-Bill with a redemption value of 1000 is priced at 985. Determine the price of a 180-day Canadian T-Bill with a redemption value of 1000.

(A) 969.12

(B) 969.96

(C) 970.48

(D) 971.26

(E) 971.52

C: $P\left(1+i\cdot\frac{180}{365}\right)=1000$

US: $985=1000\left(1-d\cdot\frac{180}{360}\right) \Rightarrow d=.03$

$i=2d \Rightarrow i=.06$

$\therefore \quad P=\frac{1000}{1+.06\left(\frac{180}{365}\right)}=971.26$

4. Account A credits interest using a simple discount rate of 5%. Account B credits interest using a simple interest rate, i. At time $t=3$, the forces of interest in the two accounts are equal. If 400 is deposited into account B at time $t=2$, how much will be in the account at time $t=5$.

(A) 475

(B) 480

(C) 485

(D) 490

(E) 495

$A: \ \delta_t=\frac{.05}{1-.05t}$

$\delta_3=\frac{.05}{.85}=\frac{5}{85}$

$B: \ \delta_t=\frac{i}{1+i\cdot t}$ $\quad a(t)=1+i\cdot t$

$\delta_3=\frac{i}{1+3i}$

$\therefore \quad \frac{i}{1+3i}=\frac{5}{85} \Longrightarrow i=\frac{5}{70}$

$AV^B=400\cdot paf_2^5=400\cdot\frac{a(5)}{a(2)}=400\,\frac{1+\frac{5}{70}(5)}{1+\frac{5}{70}(2)}=475$

5. Given a force of interest $\delta_t = \frac{3t}{2+t^2}$, determine the ratio $\frac{i_4}{d_3}$ where i_4 is the equivalent annual effective interest rate for the fourth year, and d_3 is the equivalent annual effective discount rate for the third year.

(A) 1.56

(B) 1.65

(C) 1.74

(D) 1.83

(E) 1.92

$$\delta_t = \frac{3}{2} \cdot \frac{2t}{2+t^2} \Rightarrow a(t) = \left(\frac{2+t^2}{2}\right)^{3/2} = \left(1+\frac{t^2}{2}\right)^{3/2}$$

$$i_4 = \frac{a(4)-a(3)}{a(3)} \qquad d_3 = \frac{a(3)-a(2)}{a(3)}$$

$$\therefore \frac{i_4}{d_3} = \frac{a(4)-a(3)}{a(3)-a(2)} = \frac{9^{3/2} - 5.5^{3/2}}{5.5^{3/2} - 3^{3/2}} = 1.83\ldots$$

6. An account credits interest using a simple discount rate, *d*, for the first half of the first year, and a discount rate of *d* compounded semiannually thereafter. A deposit of 8649 accumulates to 10,000 after 1.5 years. Determine *d*.

(A) 7.55%

(B) 8.10%

(C) 9.45%

(D) 11.85%

(E) 14.00%

$$10000 = 8649 \cdot paf_0^{1.5}$$

$$= 8649 \cdot paf_0^{0.5} \cdot paf_{0.5}^{1.5}$$

$$\therefore 10000 = 8649 \cdot (1 - d \cdot \tfrac{1}{2})^{-1} \cdot (1 - \tfrac{d}{2})^{-2}$$

$$\therefore \left(1-\frac{d}{2}\right)^3 = \frac{8649}{10000} = 0.8649$$

$$\Rightarrow d = 0.09445\ldots$$

7. An account credits interest using a simple discount rate of 5%. Let r denote the equivalent constant force of interest for the 3rd year, and let s denote the equivalent constant force of interest for the 5th year. Determine the ratio $\frac{r}{s}$.

$a(t) = (1-.05t)^{-1}$

(A) 0.79

3rd year: $paf_2^3 = \frac{a(3)}{a(2)} = \frac{90}{85} = e^r$

(B) 0.89

(C) 0.99

5th year: $paf_4^5 = \frac{a(5)}{a(4)} = \frac{.80}{.75} = e^s$

(D) 1.09

(E) 1.19

$\therefore r = \ln\left(\frac{90}{85}\right) \qquad s = \ln\left(\frac{80}{75}\right)$

$\Rightarrow \frac{r}{s} = 0.8856\ldots$

8. Sue deposits 27,000 into an account. After $3n$ years, the value of the deposit is 64,000. Rodney deposits 9,000 into another account. After $2n$ years the value of Rodney's deposit is Y. Both accounts earn the same nominal interest rate, compounded quarterly. Determine Y.

Let v = adf

(A) 14,000

$27000 = 64000\, v^{3n} \Rightarrow v^n = \frac{3}{4}$

(B) 16,000

(C) 18,000

$Y = 9000\, v^{-2n} = 9000\left(\frac{3}{4}\right)^{-2} = 16000$

(D) 20,000

(E) 22,000

9. Using an annual effect discount rate of 5%, the present value of payments of 10000 at the end of n years and another 10000 at the end of $2n$ years is 7255. Determine n.

(A) 10

(B) 11

(C) 12

(D) 13

(E) 14

$v = .95 = adf$

$7255 = 10000v^n + 10000v^{2n}$

(quadratic w/ $a = b = 10000$, $c = -7255$)

$\therefore v^n = \dfrac{-10000 \pm \sqrt{390,200,000}}{2(10000)} = 0.4876\cdots$

$\Rightarrow .95^n = 0.4876\cdots \Rightarrow n = 14$

10. An account credits interest using a simple interest rate of 7% for the first year, a nominal interest rate of 7% compounded semiannually for the second year, a nominal discount rate of 7% compounded quarterly for the third year, and a force of interest equal to 7% thereafter. Determine the accumulated value after 5 years of an initial deposit of 1000 into the account.

(A) less than 1300

(B) greater than or equal to 1300, but less than 1335

(C) greater than or equal to 1335, but less than 1370

(D) greater than or equal to 1370, but less than 1405

(E) greater than or equal to 1405

$AV = 1000 \cdot paf_0^1 \cdot paf_1^2 \cdot paf_2^3 \cdot paf_3^5$

$= 1000\,(1.07)(1.035)^2 \cdot (1 - \frac{.07}{4})^{-4} \cdot (e^{.07})^2$

$\therefore AV = 1414.93$