MAP 4170
Test 1

Name: KEY

Date: February 1, 2022

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Cathy is to receive payments of 3000 in 3 years and 5000 in 6 years. Using a nominal interest rate of $i$, compounded quarterly, the present value of the payments is 7622. Using the same interest rate, if Cathy invested 2000 for 4 years, the accumulated value would be $Y$. Determine $Y$.

   (A) 2070
   (B) 2080
   (C) 2090
   (D) 2100
   (E) 2110

Let $v$ = 3-year compounding df

$$\Rightarrow \quad 7622 = 3000v + 5000v^2$$

$$\left.\begin{array}{l} a = 5000 \\ b = 3000 \\ c = -7622 \end{array}\right\} \Rightarrow v = 0.97059\cdots$$

$$Y = 2000 \cdot (\text{4-year af}) = 2000 \cdot (\text{3-year af})^{4/3}$$

$$= 2000(v^{-1})^{4/3}$$

$$\therefore \; Y = 2000v^{-4/3} = 2081.21$$

2. An account credits interest using a force of interest $\delta_t = 0.02t + 0.01$ where $t$ is the number of years after 1/1/2020. If 1000 is deposited into the account on 1/1/2020, determine the amount of interest earned on this deposit during calendar year 2021.

   (A) 40.81
   (B) 41.21
   (C) 41.64
   (D) 60.59
   (E) 61.84

$$I_{2021} = 1000 \cdot a(2) - 1000\, a(1)$$

$$a(n) = e^{\int_0^n (.02t + .01)\,dt} = e^{(.01t^2 + .01t)\big|_0^n} = e^{.01n^2 + .01n}$$

$$\therefore \; a(2) = e^{.06}$$

$$a(1) = e^{.02}$$

$$\Rightarrow I_{2021} = 1000e^{.06} - 1000e^{.02} = 41.64$$

3. The quoted rate for a 90-day Canadian T-Bill is half the quoted rate for a 180-day U.S. T-Bill. A 90-day Canadian T-Bill with a redemption value of 1000 is priced at 990. To the nearest dollar, determine the price of a 180-day U.S. T-Bill with a redemption value of 1000.

(A) 957

(B) 958

(C) 959

(D) 960

(E) 961

C: $990\left(1+i\cdot\frac{90}{365}\right) = 1000 \Rightarrow i = 0.04096\ldots$

US: $P = 1000\left(1-d\cdot\frac{180}{360}\right)$ $\quad d = 2i$ (given)

$\therefore P = 1000\left(1-2i\cdot\frac{1}{2}\right)$

$= 1000(1-i) = 959.03$

4. Account $A$ credits interest using a simple discount rate of 5%. Account $B$ credits interest using a quarterly effective interest rate, $i$. At time $t = 2$, the forces of interest in the two accounts are equal. Determine the amount that needs to be deposited into Account $B$ in order for there to be 1000 in the account after 3 years.

(A) 825

(B) 830

(C) 835

(D) 840

(E) 845

A: $d = .05$ simple $\Rightarrow \delta_t = \frac{.05}{1-.05t}$

$\delta_2 = \frac{.05}{1-.05(2)} = \frac{5}{90}$

B: compounding $\Rightarrow \delta_t = \delta = \ln(aaf)$

$t=2 \Rightarrow \frac{5}{90} = \ln(aaf) \Rightarrow aaf = e^{5/90}$

Let $X$ = amount in question.

$\therefore X = 1000\,(\text{3-year } df) = 1000\cdot(adf)^3 = 1000\left(e^{-5/90}\right)^3$

$\Rightarrow X = 1000e^{-15/90} = 846.48$

5. Given a force of interest $\delta_t = \frac{kt}{2+t^2}$, an amount of 1000 at time $t = 5$ discounts to 800 at time $t = 1$. Determine the equivalent value at time $t = 3$.

(A) 887

(B) 894

(C) 900

(D) 906

(E) 913

$$\cancel{\delta_t = \frac{k}{2}\cdot\frac{2t}{2+t^2}} \quad \delta_t = \frac{k}{2}\cdot\frac{2t}{2+t^2} \Rightarrow a(t) = \left(\frac{2+t^2}{2}\right)^{k/2} = \left(1+\frac{t^2}{2}\right)^{k/2}$$

$$800 = 1000\cdot\frac{a(1)}{a(5)} = 1000\cdot\frac{(1.5)^{k/2}}{(13.5)^{k/2}} = 1000\left(\frac{15}{135}\right)^{k/2}$$

$$\Rightarrow 0.8 = \left(\frac{15}{135}\right)^{k/2} \Rightarrow \frac{k}{2} = \frac{\ln(0.8)}{\ln\left(\frac{15}{135}\right)} = 0.101557\cdots$$

The time 3 value $= 1000\cdot\frac{a(3)}{a(5)} = 1000\cdot\frac{(5.5)^{k/2}}{(13.5)^{k/2}}$

$$= 1000\left(\frac{55}{135}\right)^{k/2} = 912.84$$

6. An account credits interest using a simple interest rate, $i$, for the first quarter of the first year, and an interest rate of $i$ compounded quarterly thereafter. An initial deposit of 1000 accumulates to 1325 after 2.5 years. Determine the amount the initial deposit has accumulated to after 5 years.

(A) 1600

(B) 1650

(C) 1700

(D) 1750

(E) 1800

$a(t) = 1 + it$ | seir $= i/4$, saf $= 1 + \frac{i}{4} = 1 + .25i$

yrs 0 0.25

$$1325 = 1000\cdot \underline{paf_0^{0.25}}\cdot \underline{\underline{paf_{0.25}^{2.5}}} = 1000\underline{(1+.25i)}\cdot\underline{\underline{(1+.25i)^9}}$$

$$\Rightarrow (1+.25i)^{10} = 1.325$$

After 5 years, we have

$$AV = 1000\cdot paf_0^{0.25}\cdot paf_{0.25}^{5} = 1000(1+.25i)\cdot(1+.25i)^{19}$$

$$= 1000(1+.25i)^{20} = 1000(1.325)^2 = 1755.63$$

7. An account credits interest using a simple interest rate of 5%. Determine the ratio $\frac{i_5}{d_5}$, where $i_5$ is the equivalent annual effective interest rate for the 5th year and $d_5$ is the equivalent annual effective discount rate for the 5th year.

(A) $\frac{23}{24}$

(B) $\frac{24}{25}$

(C) 1

(D) $\frac{25}{24}$

(E) $\frac{24}{23}$

$i = .05$ simple $\Rightarrow a(t) = 1 + .05t$

$$i_5 = \frac{a(5) - a(4)}{a(4)} \qquad d_5 = \frac{a(5) - a(4)}{a(5)}$$

$$\therefore \frac{i_5}{d_5} = \frac{a(5)}{a(4)} = \frac{1 + .05(5)}{1 + .05(4)} = \frac{125}{120} = \frac{25}{24}$$

8. For an account that credits interest using a nominal interest rate of $i$, compounded monthly, a deposit 14,000 accumulates to 126,000 after $2k$ years. Determine the amount needed to be deposited into this account in order for the deposit to accumulate to 45,000 after $k$ years.

(A) 15,000

(B) 16,000

(C) 17,000

(D) 18,000

(E) 19,000

compounding

$126000 = 14000 \cdot (2k\text{-year af})$

$\Rightarrow$ ~~2k~~ $2k\text{-year af} = \frac{126}{14} = 9$

Let $X$ = the amount in question

$X = 45000 \cdot (k\text{-year df})$

$2k\text{-year af} = 9 \Rightarrow k\text{-year af} = 9^{1/2} = 3 \Rightarrow k\text{-year df} = \frac{1}{3}$

$\therefore X = 45000 \cdot \left(\frac{1}{3}\right) = 15000$

9. Judy is to receive payments of $X$ in 1 year and $2X$ in 2 years. Jon is to receive payments of $X$ in 3 years and $2X$ in 4 years. Using an annual effective discount rate of $d$, the present value of Jon's payments is equal to 81% of the present value of Judy's payments. Determine $d$. Let $v = 1 - d =$ adf

(A) 10%

(B) 11%

(C) 12%

(D) 13%

(E) 14%

Jon: $PV = Xv^3 + 2Xv^4$

Judy: $PV = Xv + 2Xv^2$

Jon's PV $= 0.81 \cdot$ Judy's PV

$$\Rightarrow \underbrace{Xv^3 + 2Xv^4}_{= v^2(Xv + 2Xv^2)} = 0.81(Xv + 2Xv^2)$$

$$\therefore \quad v^2 = 0.81 \Rightarrow v = 0.9$$

$$\therefore \quad d = 1 - v = 0.1$$

10. An account credits interest using a simple discount rate of 3% for the first two years, a nominal discount rate of 4% compounded biannually for the third year, and a force of interest equal to 5% thereafter. Determine the equivalent annual effective interest rate over the first 5-year period.

(A) less than 4.13%

(B) greater than or equal to 4.13%, but less than 4.18%

(C) greater than or equal to 4.18%, but less than 4.23%

(D) greater than or equal to 4.23%, but less than 4.28%

(E) greater than or equal to 4.28%

beir = .08

$d = .03$ simple | baf $= (.92)^{-1}$ | $\delta = .05$

Yrs 0 | 2 | 3 | 5

$$a(5) = (1 - .03(2))^{-1} \cdot (.92^{-1})^{1/2} \cdot e^{.05(2)} = (.94)^{-1} \cdot (.92^{-1/2}) \cdot e^{.1} = 1.22576\ldots$$

Using $i =$ aeir, $a(5) = (1+i)^5$

$$\therefore (1+i)^5 = 1.22576\cdots \Rightarrow i = 0.041553\cdots$$