

MAP 4170
Test 1

Name: KEY

Date: February 1, 2022

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Cathy is to receive payments of 3000 in 3 years and 5000 in 6 years. Using a nominal interest rate of i , compounded quarterly, the present value of the payments is 7622. Using the same interest rate, if Cathy invested 2000 for 4 years, the accumulated value would be Y . Determine Y .

(A) 2070

(B) 2080

(C) 2090

(D) 2100

(E) 2110

compounding

Let $v = 3\text{-year df}$

$$\Rightarrow 7622 = 3000v + 5000v^2$$

$$\left. \begin{array}{l} a = 5000 \\ b = 3000 \\ c = -7622 \end{array} \right\} \Rightarrow v = 0.97059\dots$$

$$Y = 2000 \cdot (4\text{-year af}) = 2000 \cdot (3\text{-year af})^{4/3}$$

$$= 2000(v^{-1})^{4/3}$$

$$\therefore Y = 2000v^{-4/3} = 2081.21$$

2. An account credits interest using a force of interest $\delta_t = 0.02t + 0.01$ where t is the number of years after 1/1/2020. If 1000 is deposited into the account on 1/1/2020, determine the amount of interest earned on this deposit during calendar year 2021.

(A) 40.81

(B) 41.21

(C) 41.64

(D) 60.59

(E) 61.84

$$I_{2021} = 1000 \cdot a(2) - 1000 a(1)$$

$$a(n) = e^{\int_0^n (0.02t + 0.01) dt} = e^{(0.01t^2 + 0.01t)|_0^n} = e^{.01n^2 + .01n}$$

$$\therefore a(2) = e^{.06}$$

$$a(1) = e^{.02}$$

$$\Rightarrow I_{2021} = 1000e^{.06} - 1000e^{.02} = 41.64$$

3. The quoted rate for a 90-day Canadian T-Bill is half the quoted rate for a 180-day U.S. T-Bill. A 90-day Canadian T-Bill with a redemption value of 1000 is priced at 990. To the nearest dollar, determine the price of a 180-day U.S. T-Bill with a redemption value of 1000.

(A) 957 C: $990 \left(1 + i \cdot \frac{90}{365}\right) = 1000 \implies i = 0.04096 \dots$

(B) 958

(C) 959 US: $P = 1000 \left(1 - d \cdot \frac{180}{360}\right)$ $d = 2i$ (given)

(D) 960

(E) 961 $\therefore P = 1000 \left(1 - 2i \cdot \frac{1}{2}\right)$

$$= 1000(1 - i) = 959.03$$

4. Account A credits interest using a simple discount rate of 5%. Account B credits interest using a quarterly effective interest rate, i . At time $t = 2$, the forces of interest in the two accounts are equal. Determine the amount that needs to be deposited into Account B in order for there to be 1000 in the account after 3 years.

(A) 825 A: $d = .05$ simple $\implies \delta_t = \frac{.05}{1 - .05t}$

(B) 830 $\delta_2 = \frac{.05}{1 - .05(2)} = \frac{5}{90}$

(C) 835

(D) 840 B: compounding $\implies \delta_t = \delta = \ln(aaf)$

(E) 845 $t = 2 \implies \frac{5}{90} = \ln(aaf) \implies aaf = e^{5/90}$

Let X = amount in question.

$$\therefore X = 1000 \text{ (3-year df)} = 1000 \cdot (adf)^3 = 1000 \left(e^{-5/90}\right)^3$$

$$\implies X = 1000 e^{-15/90} = 846.48$$

5. Given a force of interest $\delta_t = \frac{kt}{2+t^2}$, an amount of 1000 at time $t = 5$ discounts to 800 at time $t = 1$. Determine the equivalent value at time $t = 3$.

(A) 887

(B) 894

(C) 900

(D) 906

(E) 913

$$\delta_t = \frac{kt}{2+t^2} \Rightarrow \int_t^s \delta_t dt = \frac{k}{2} \cdot \frac{2t}{2+t^2} \Rightarrow a(t) = \left(\frac{2+t^2}{2}\right)^{k/2} = \left(1 + \frac{t^2}{2}\right)^{k/2}$$

$$800 = 1000 \cdot \frac{a(1)}{a(5)} = 1000 \cdot \frac{(1.5)^{k/2}}{(13.5)^{k/2}} = 1000 \left(\frac{15}{135}\right)^{k/2}$$

$$\Rightarrow 0.8 = \left(\frac{15}{135}\right)^{k/2} \Rightarrow \frac{k}{2} = \frac{\ln(0.8)}{\ln\left(\frac{15}{135}\right)} = 0.101557 \dots$$

$$\text{The time 3 value} = 1000 \cdot \frac{a(3)}{a(5)} = 1000 \cdot \frac{(5.5)^{k/2}}{(13.5)^{k/2}}$$

$$= 1000 \left(\frac{55}{135}\right)^{k/2} = 912.84$$

6. An account credits interest using a simple interest rate, i , for the first quarter of the first year, and an interest rate of i compounded quarterly thereafter. An initial deposit of 1000 accumulates to 1325 after 2.5 years. Determine the amount the initial deposit has accumulated to after 5 years.

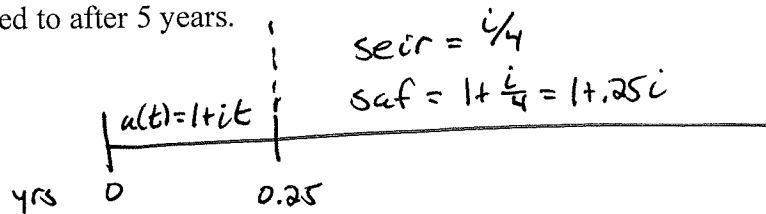
(A) 1600

(B) 1650

(C) 1700

(D) 1750

(E) 1800



$$1325 = 1000 \cdot \text{paf}_0^{0.25} \cdot \text{paf}_{0.25}^{2.5} = 1000 \cdot \underline{(1.25i)} \cdot \underline{(1.25i)^9}$$

$$\Rightarrow (1.25i)^{10} = 1.325$$

After 5 years, we have

$$AV = 1000 \cdot \text{paf}_0^{0.25} \cdot \text{paf}_{0.25}^5 = 1000 \cdot (1.25i) \cdot (1.25i)^9$$

$$= 1000 (1.25i)^{10} = 1000 (1.325)^2 = 1755.63$$

7. An account credits interest using a simple interest rate of 5%. Determine the ratio $\frac{i_5}{d_5}$, where i_5 is the equivalent annual effective interest rate for the 5th year and d_5 is the equivalent annual effective discount rate for the 5th year.

(A) $\frac{23}{24}$

$$i = .05 \text{ simple} \implies a(t) = 1 + .05t$$

(B) $\frac{24}{25}$

$$i_5 = \frac{a(5) - a(4)}{a(4)} \quad d_5 = \frac{a(5) - a(4)}{a(5)}$$

(C) 1

(D) $\frac{25}{24}$

$$\therefore \frac{i_5}{d_5} = \frac{a(5)}{a(4)} = \frac{1 + .05(5)}{1 + .05(4)} = \frac{1.25}{1.20} = \frac{25}{24}$$

(E) $\frac{24}{23}$

8. For an account that credits interest using a nominal interest rate of i , compounded monthly, a deposit 14,000 accumulates to 126,000 after $2k$ years. Determine the amount needed to be deposited into this account in order for the deposit to accumulate to 45,000 after k years.

(A) 15,000

compounding

$$126000 = 14000 \cdot (2k\text{-year af})$$

(B) 16,000

$$\implies \cancel{2k}\text{-year af} = \frac{126}{14} = 9$$

(C) 17,000

(D) 18,000

Let X = the amount in question

(E) 19,000

$$X = 45000 \cdot (k\text{-year df})$$

$$2k\text{-year af} = 9 \implies k\text{-year af} = 9^{1/2} = 3 \implies k\text{-year df} = \frac{1}{3}$$

$$\therefore X = 45000 \cdot \left(\frac{1}{3}\right) = 15000$$

9. Judy is to receive payments of X in 1 year and $2X$ in 2 years. Jon is to receive payments of X in 3 years and $2X$ in 4 years. Using an annual effective discount rate of d , the present value of Jon's payments is equal to 81% of the present value of Judy's payments. Determine d . Let $v = 1 - d = adf$

(A) 10%

Jon: $PV = Xv^3 + 2Xv^4$

(B) 11%

Judy: $PV = Xv + 2Xv^2$

(C) 12%

Jon's $PV = 0.81 \cdot \text{Judy's } PV$

(D) 13%

(E) 14%

$$\Rightarrow \underbrace{Xv^3 + 2Xv^4}_{= v^2(Xv + 2Xv^2)} = 0.81(Xv + 2Xv^2)$$

$$\therefore v^2 = 0.81 \Rightarrow v = 0.9$$

$$\therefore d = 1 - v = 0.1$$

10. An account credits interest using a simple discount rate of 3% for the first two years, a nominal discount rate of 4% compounded biannually for the third year, and a force of interest equal to 5% thereafter. Determine the equivalent annual effective interest rate over the first 5-year period.

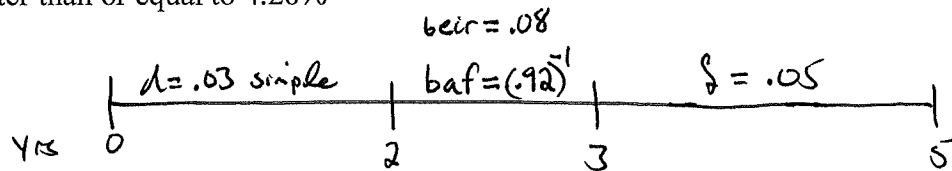
(A) less than 4.13%

(B) greater than or equal to 4.13%, but less than 4.18%

(C) greater than or equal to 4.18%, but less than 4.23%

(D) greater than or equal to 4.23%, but less than 4.28%

(E) greater than or equal to 4.28%



$$a(5) = (1 - .03(2))^{-1} \cdot (.92)^{-1/2} \cdot e^{.05(2)} = (.94)^{-1} \cdot (.92)^{-1/2} \cdot e^1 = \frac{1.22576...}{.94} = 1.295...$$

Using $i = aeir$, $a(5) = (1+i)^5$

$$\therefore (1+i)^5 = 1.22576... \Rightarrow i = 0.041553...$$