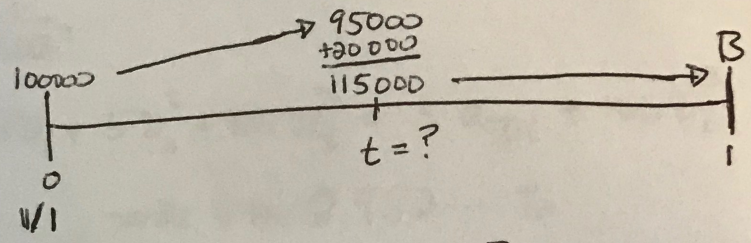
MAP4170 Test 4 Key:

1) $i' = \frac{1.09}{1.03} - 1 = 5,825\cdots\%$

$PV = 1000\, a_{\overline{20}|i'} = 11,634.43$

2)



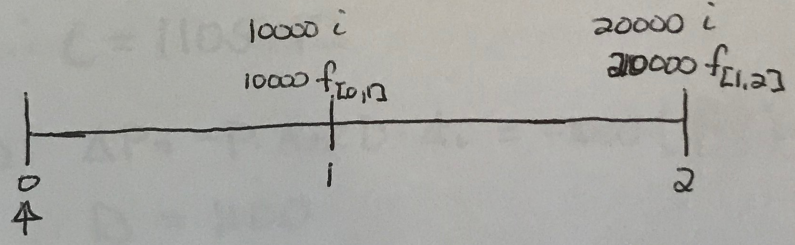
$i_{TW} = .14 \Rightarrow 1.14 = \frac{95000}{100000} \cdot \frac{B}{115000} \Rightarrow B = 138000$

$i_{DW} = .17 \Rightarrow 100000(1.17) + 20000(1 + .17(1-t)) = B = 138000$

$\Rightarrow t = 0.705\,88\cdots$ yrs $= 8.47\cdots$ months

8.47... months after Jan. 1 is in September

3)



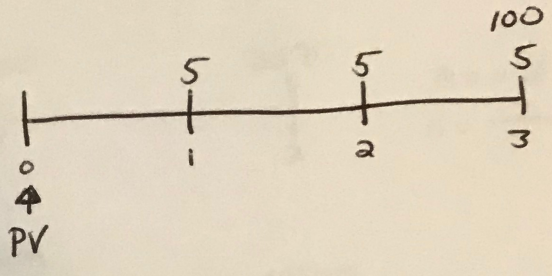
$10000i \cdot v_1 + 20000i \cdot v_2^2 = 10000 \underbrace{f_{[0,1]} \cdot v_1}_{= 1 - v_1} + 20000 \underbrace{f_{[1,2]} \cdot v_2^2}_{= v_1 - v_2^2}$

$\Rightarrow i = \frac{10000(1-v_1) + 20000(v_1 - v_2^2)}{10000 v_1 + 20000 v_2^2}$

$v_1 = \frac{1}{1.035} \qquad v_2^2 = \frac{1}{1.038^2}$

$\therefore \quad i = 0.03895\cdots$

4) With $F=100$,



$v_1 = \frac{1}{1.02}$

$v_2^2 = \frac{1}{1.02^2}$

$v_3^3 = \frac{1}{1.03^3}$

$PV = 5v_1 + 5v_2^2 + 105v_3^3 = 5\,a_{\overline{3}|i} + 100v_i^3$ (i = annual yield)

$\Rightarrow i = 2.952\cdots\%$

5) $(1+s_4)^4 = (1+s_2)^2 \cdot (1+f_{[2,4]})^2$

$s_4 = i_4 = .055 \qquad s_2 = i_2 = .045 \Rightarrow f_{[2,4]} = 0.06509\cdots$

6) For C: $\Delta P = P\left[\left(\frac{1+i_{old}}{1+i_{new}}\right)^{MacD} - 1\right] = 1000\left[\left(\frac{1.07}{1.065}\right)^{21.4} - 1\right] = 105.43$

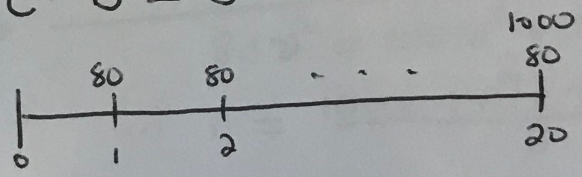
$\therefore C = 1105.43$

For D: $\Delta P = -P \cdot Mod\,D \cdot \Delta i = -1000\left(\frac{21.4}{1.07}\right)\cdot(.065-.07) = 100$

$\therefore D = 1100$

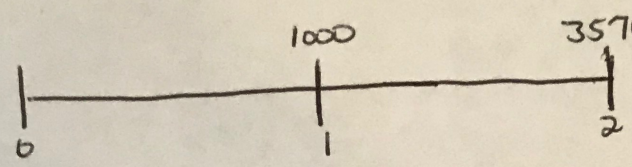
$\therefore C - D = 5.43$

7)



$$MacD = \frac{80\cdot(Ia)_{\overline{20}|.06} + 20\cdot 1000v_{.06}^{20}}{80\,a_{\overline{20}|.06} + 1000v_{.06}^{20}} = 11.495\cdots$$

8) L:



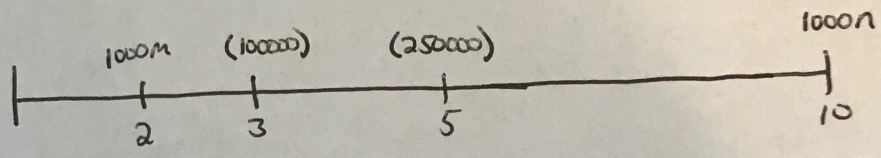
m = # of 1-year bonds
n = —— 2-year ——

A:

$100m$
$50n$
$1050n$

$$\therefore \begin{cases} 100m + 50n = 1000 \\ 1050n = 3570 \end{cases} \Rightarrow \begin{array}{c} n = 3.4 \\ \boxed{m = 8.3} \end{array}$$

9) Let m = # of 2-year bonds
n = —— 10-year ——



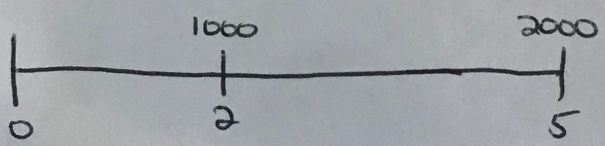
$$\begin{cases} 1000m \cdot v^2 + 1000n \cdot v^{10} = 100000v^3 + 250000v^5 \\ 2 \cdot 1000m \cdot v^2 + 10 \cdot 1000n \cdot v^{10} = 3 \cdot 100000v^3 + 5 \cdot 250000v^5 \end{cases}$$

$$\begin{cases} (m \cdot v^2 + nv^{10} = 100v^3 + 250v^5)(-2) \\ 2mv^2 + 10nv^{10} = 300v^3 + 1250v^5 \end{cases}$$

$$+ \quad 8nv^{10} = 100v^3 + 750v^5$$

$$\therefore n = \frac{100(1.04)^7 + 750(1.04)^5}{8} = 130.51\cdots$$

10)



$v = \frac{1}{1.05}$

$$\text{Mac}\,C = \frac{2^2 \cdot 1000v^2 + 5^2 \cdot 2000v^5}{1000v^2 + 2000v^5} = 17.30\cdots$$