MAP 4170 Name:____________________________
Test 4 Date: November 30, 2021

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. The relevant characteristics of two bonds forming a portfolio are:

(i) Bond A has a Macaulay duration of 5 years and a purchase price of X.

(ii) Bond B has a Macaulay duration of 15 years and a purchase price of $1.5X$.

For both bonds the Macaulay duration and price were determined using an annual effective interest rate of 10%. Using the same interest rate, determine the Modified duration of the portfolio.

weights

$P^{portfolio} = X + 1.5X = 2.5X$ $\quad \frac{X}{2.5X} = \frac{1}{2.5}$ & $\frac{1.5X}{2.5X} = \frac{1.5}{2.5}$

$\therefore MacD^{portfolio} = \frac{1}{2.5}(5) + \frac{1.5}{2.5}(15) = 11$

$\Rightarrow ModD^{portfolio} = v \cdot MacD^{portfolio} = \frac{11}{1.1} = 10$

A) 9.1

B) 10.0

C) 11.0

D) 12.1

E) 13.3

2. Assuming an annual inflation rate of 2% and a nominal annual rate of 9%, use the real rate of return to determine the accumulated value of a 15-year annuity due with annual payments of 30,000.

real rate of return $= \frac{1.09}{1.02} - 1 = 0.0686\ldots = i'$

$AV = 30000\, \ddot{s}_{\overline{15}|i'} = 797,141$

A) 695,000

B) 720,000

C) 745,000

D) 770,000

E) 795,000

3. An account has a balance of 500,000 at the beginning of the year. During the year, there is a contribution of 50,000 made into the account. The balance immediately after the contribution is 600,000. The balance at the end of the year is 720,000. Given that the time-weighted and dollar-weighted interest rates for the year are equal, determine the month in which the contribution is made.

A) April

B) May

C) July

D) August

E) September

500000 → 550000 + 50000 = 600000 → 720000

0 ——— t ——— 1

$$1+i_{TW} = \frac{550000}{500000}\cdot\frac{720000}{600000} = 1.32 \Rightarrow i_{TW} = i_{DW} = 0.32$$

$$\therefore\ 500000(1.32) + 50000\,(1+0.32(1-t)) = 720000$$

$$\Rightarrow t = 0.375 \text{ (years)}$$

$$\overset{\times 12}{=} 4.5 \text{ (months) after } 1/1$$

$\therefore$ The contribution is made in May

4. At a 6% annual effective interest rate, the price of a bond is 1200 and its Macaulay duration is 15. Using the first order modified approximation, determine the approximate price of the bond if the interest rate is changed to 6.5%.

A) Less than 1107

B) Greater than or equal to 1107 but less than 1112

C) Greater than or equal to 1112 but less than 1117

D) Greater than or equal to 1117 but less than 1122

E) Greater than or equal to 1122

$$\text{Mod}D_{.06} = \frac{15}{1.06}$$

$$\Delta P = -P\cdot \text{Mod}D\cdot \Delta i$$

$$= -1200\cdot\frac{15}{1.06}\cdot(.005) = -84.91$$

$$\therefore\ P(.065) \approx 1200 - 84.91 = 1115.09$$

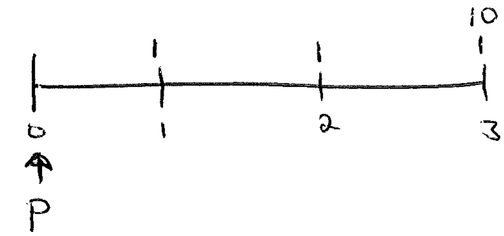
For Numbers 5 and 6:
You are given the following prices on zero coupon bonds redeemable for 1000:

MacD (in years)	Price
1	950
2	890
3	830
4	775

$\Rightarrow$ $v_1 = 0.950$, $v_2^2 = 0.890$, $v_3^3 = 0.830$, $v_4^4 = 0.775$

5. Determine the annual yield on 3-year 10% annual coupon bonds that is consistent with the above bond prices.

With $F = 10$:



A) 6.29%

B) 6.31%

C) 6.33%

D) 6.35%

E) 6.37%

$$P = v_1 + v_2^2 + 11 v_3^3 = a_{\overline{3}|i} + 10 v_i^3$$

$$\Rightarrow i = 0.063477\ldots$$

6. Determine the 1-year forward rate for year 4 (from time 3 to time 4) implied by this yield curve.

A) 6.3%

B) 6.5%

C) 6.7%

D) 6.9%

E) 7.1%

$$(1+s_3)^3 (1 + f_{[3,4]}) = (1+s_4)^4$$

$$(1+s_3)^3 = v_3^{-3} = (.83)^{-1}$$

$$(1+s_4)^4 = v_4^{-4} = (.775)^{-1}$$

$$\therefore\ 1 + f_{[3,4]} = \frac{830}{775} \Rightarrow f_{[3,4]} = 0.07096\ldots$$

7. An n-year bond, redeemable at par, with 3.5% annual coupons has a Macaulay duration of 19.392 years when using an annual effective yield rate of 3.5%. Determine n.

(A) 31

(B) 32

(C) 33

(D) 34

(E) 35

Since $F = R$ and $i = r = 0.035$

then $MacD = \ddot{a}_{\overline{n}|}$

$\therefore\ 19.392 = \ddot{a}_{\overline{n}|.035} \Rightarrow n = 31$

8. Payments of 100, 110, and 121 are to be made at the beginning of years 1, 2, and 3, respectively. Determine the modified convexity of this sequence of payments using an annual effective interest rate of 10%.

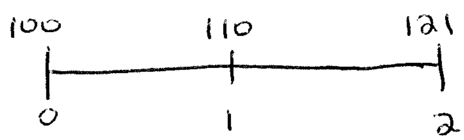
(A) 2.0

(B) 2.1

(C) 2.2

(D) 2.3

(E) 2.4



$i = 0.1$

$$P(i) = 100 + 110(1+i)^{-1} + 121(1+i)^{-2}$$

$$P'(i) = -110(1+i)^{-2} - 242(1+i)^{-3}$$

$$P''(i) = 220(1+i)^{-3} + 726(1+i)^{-4}$$

$$ModC_{0.1} = \frac{P''(0.1)}{P(0.1)} = \frac{220(1.1)^{-3} + 726(1.1)^{-4}}{100 + 110(1.1)^{-1} + 121(1.1)^{-2}} = 2.20\ldots$$

9. Using an interest rate such that the periodic discount factor, v, equals 0.9, a liability payment of X at time n ($1 < n < 4$) is to be immunized using assets that pay 5,000 at time 1 and 10,000 at time 4. Determine X.

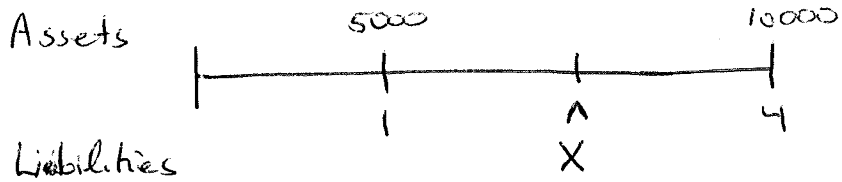
A) 14,750

B) 14,775

C) 14,800

D) 14,825

E) 14,850



$\therefore$ (1) $5000v + 10000v^4 = Xv^n$

(2) $5000v + 40000v^4 = n \cdot X v^n$

Since $v = 0.9$, equation (1) $\Rightarrow X \cdot v^n = 5000(.9) + 10000(.9)^4$

$\Rightarrow X v^n = 11061$

Plugging into equation (2), we get $5000(.9) + 40000(.9)^4 = n \cdot (11061)$

$\Rightarrow n = 2.779\ldots$

$\therefore 5000(.9) + 10000(.9)^4 = X(.9)^{2.779\ldots} \Rightarrow X = 14824.40$

10. For k = 1, 2, 3, 4, and 5, liabilities of $2600 - 100k$ at time k are to be exactly matched using k-year 100 face value bonds, redeemable at par, with 5% annual coupons. All the bonds can be bought to yield 6% annual effective. Determine the total cost of buying all the bonds needed for the exact matching of the liabilities.

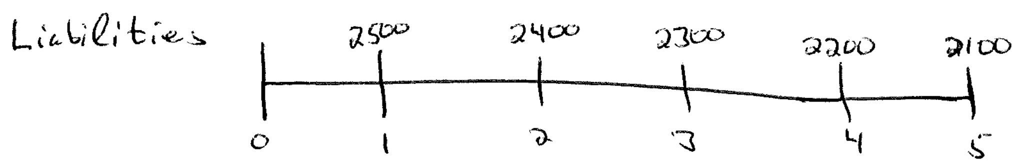
(A) 9740

(B) 9750

(C) 9760

(D) 9770

(E) 9780



Since all the bonds are bought at the same yield of 0.06 aeir, and the liabilities are exactly matched with bond payments, then the $\underbrace{\text{total cost}}_{TC}$ is the PV of the liabilities.

$$(Da)_{\overline{5|}} = \frac{5 - a_{\overline{5|}}}{i}$$

$$\therefore \; TC = 2000\, a_{\overline{5|}} + 100 \cdot (Da)_{\overline{5|}}$$

$$\overset{i=.06}{=} 9737.45$$