MAP 4170 Name:________________________________

Test 4 April 21, 2020

Each problem is worth 10 points. Show sufficient work for full credit.

1. Angela and Pam are each asked to calculate the convexity of a 3-year zero coupon bond. Angela calculates the modified convexity and gets the answer, A, while Pam calculates the Macaulay convexity and gets the answer, P. Both Angela and Pam used an annual effective interest rate of 10%. Calculate the ratio, P/A.

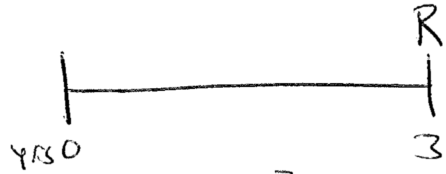
A) less than or equal to 0.8

B) greater than 0.8, but less than or equal to 0.9

C) greater than 0.9, but less than or equal to 1.0

D) greater than 1.0, but less than or equal to 1.1

E) greater than 1.1



$$P(i) = R(1+i)^{-3}$$
$$P'(i) = -3R(1+i)^{-4}$$
$$P''(i) = 12R(1+i)^{-5}$$

$$\therefore \left.\begin{array}{l} A = \dfrac{P''(.1)}{P(.1)} = \dfrac{12R(1.1)^{-5}}{R(1.1)^{-3}} = \dfrac{12}{(1.1)^2} \\ P = \dfrac{3^2 \cdot R \cdot v^3}{R \cdot v^3} = 9 \end{array}\right\} \Longrightarrow P/A = \frac{9}{\left(12/(1.1)^2\right)} = 0.9075$$

2. The one-year forward rate from time $t = 0$ to time $t = 1$ is 3.0%.
The one-year forward rate from time $t = 1$ to time $t = 2$ is 3.5%.
The one-year forward rate from time $t = 2$ to time $t = 3$ is 4.0%.
All time measurements are in years. Determine the implied yield rate on 3-year 5% annual coupon bonds.

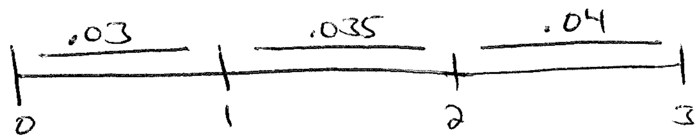
A) 3.48%

B) 3.60%

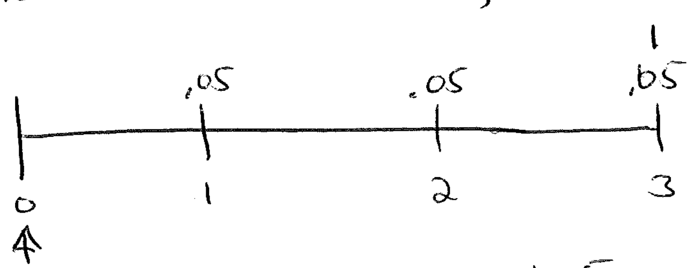
C) 3.73%

D) 3.85%

E) 3.97%



Per dollar of face value,



$$P = \frac{.05}{1.03} + \frac{.05}{(1.035)(1.03)} + \frac{1.05}{(1.04)(1.035)(1.03)} = .05a_{\overline{3}|i} + v_i^3$$

$$\xrightarrow{\text{TVM}} i = .0348\ldots$$

3. Bonds A and B are each 1000 face value 4-year bonds with annual coupons, and each bond is redeemable at par. Bond A has a 2% coupon rate and is priced at 894.16, whereas Bond B has a 5% coupon rate and is priced at 1001.33. The 1-year and 2-year spot rates are each equal to 4%. Determine the 3-year spot rate.

A) 4.0%

B) 4.5%

C) 5.0%

D) 5.5%

E) 6.0%

Bond A: $(894.16 = 20v_1 + 20v_2^2 + 20v_3^3 + 1020v_4^4)(-2.5)$

Bond B: $+\quad 1001.33 = 50v_1 + 50v_2^2 + 50v_3^3 + 1050v_4^4$

$-1234.07 = -1500v_4^4 \Rightarrow v_4^4 = \dfrac{1234.07}{1500}$

$$\therefore\ 894.16 = \frac{20}{1.04} + \frac{20}{(1.04)^2} + \frac{20}{(1+S_3)^3} + 1020\cdot\left(\frac{1234.07}{1500}\right)$$

$$\Rightarrow S_3 = 0.0501\ldots$$

4. An asset sells for \$1110 to yield 4% annual effective. If the interest rate is changed to 6%, the approximate price of the asset using the first order modified approximation is \$860. Using the first order Macaulay approximation, determine the approximate price of the asset if the interest rate is changed from 4% to 6%.

A) 855

B) 866

C) 877

D) 888

E) 899

$\left.\begin{array}{l} P(.04) = 1110 \\ P(.06) = 860 \end{array}\right\} \Rightarrow \Delta P = -250$ when $\Delta i = .02$

1st order modified: $\Delta P = -P\,\text{Mod}D\cdot\Delta i$

$\Rightarrow -250 = -1110\cdot\text{Mod}D\cdot(.02)$

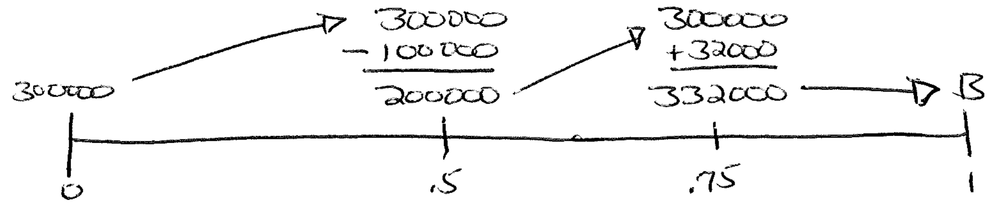
$\Rightarrow \text{Mod}D = 11.\overline{261}$ @ 4%

$\Rightarrow \text{Mac}D = \text{Mod}D\cdot(1.04) = 11.\overline{711}$ @ 4%

1st order Macaulay: $\Delta P = P\cdot\left[\left(\dfrac{1+i_{old}}{1+i_{new}}\right)^{\text{Mac}D} - 1\right]$

$= 1110\cdot\left[\left(\dfrac{1.04}{1.06}\right)^{11.\overline{711}} - 1\right] = -221.95$

$\Rightarrow P(.06) \overset{\text{Mac}D}{\approx} 1110 - 221.95 = 888.05$



5. The balance in an investment account on January 1 is 300000. On July 1, the account value is 300000 and then 100000 is withdrawn from the account, leaving 200000 in the account. On October 1, the account value is 300000 and then 32000 is deposited into the account, leaving 332000 in the account. There are no other transactions within the account during the year.

   The dollar weighted return for the year is -4% (negative 4 percent). Determine the time weighted return for the year.

   (A) less than -3%

   (B) greater than or equal to -3%, but less than -2%

   (C) greater than or equal to -2%, but less than -1%

   (D) greater than or equal to -1%, but less than 0%

   (E) greater than or equal to 0%

$$i_{DW} = -.04 \Rightarrow B = 300000(1-.04) - 100000(1-.04(.5)) + 32000(1-.04(.25))$$
$$= 221680$$
$$\therefore i_{TW} = \frac{300000}{300000} \cdot \frac{300000}{200000} \cdot \frac{221680}{332000} - 1 = .0015\ldots$$

6. Charles and Thomas are each asked to calculate the accumulated value of a 40-year annuity due with annual payments of 5000. Charles uses a nominal interest rate of 8%, compounded annually and determines the answer to be *C*. Thomas also assumes a nominal rate of 8% compounded annually, but in his calculation, he uses the real rate of return based on a 2% annual effective rate of inflation. Thomas determines the answer to be *T*. Determine the ratio *C/T*.

   A) 1.71

   B) 1.76

   C) 1.81

   D) 1.86

   E) 1.91

C: $AV = 5000\, \ddot{s}_{\overline{40}|8\%} = 1398905 = C$

T: Uses $i' = \frac{1.08}{1.02} - 1 = .0588\ldots$

$AV = 5000\, \ddot{s}_{\overline{40}|i'} = 795494 = T$

$\therefore C/T = 1.758\ldots$

7. A company has two liabilities: a payment of 2000 at the end of 2 years and a payment of 3000 at the end of 3 years. The current yield curve is flat at 5%. The company will take advantage of this economic situation by fully immunizing their liabilities using a 1-year zero coupon bond and a 4-year zero coupon bond. Determine the amount of redemption value for the 1-year zero coupon bond that the company needs in order to achieve full immunization.

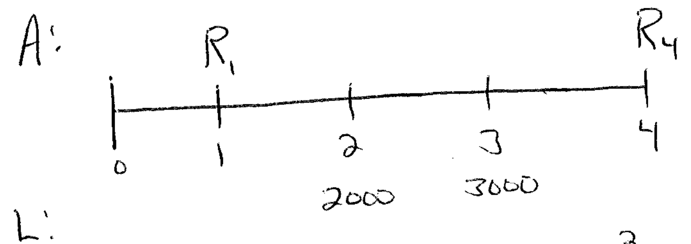
A) 2073

B) 2099

C) 2125

D) 2151

E) 2177



$v = \frac{1}{1.05}$

$$(-4)\left(R_1 v + R_4 \cdot v^4 = 2000v^2 + 3000v^3\right)$$
$$+\ R_1 v + 4R_4 v^4 = 4000v^2 + 9000v^3$$
$$-3R_1 v = -4000v^2 - 3000v^3$$
$$\Rightarrow R_1 = 2176.87$$

8. A company has a liability of 2000 at the end of one year and another liability of 3400 at the end of two years. The company will exactly match these liabilities using two assets; Asset *X* and Asset *Y*.

Asset *X* can be bought to yield 5% annual effective; it pays *x* at the end of one year and *2x* at the end of two years.

Asset *Y* can be bought to yield 6% annual effective; it pays *2y* at the end of one year and *y* at the end of two years.

Determine the total cost for the company to exactly match their liabilities using these two assets.

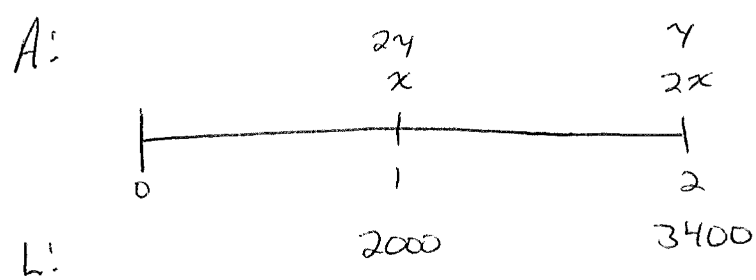
A) 4973

B) 4982

C) 4989

D) 4993

E) 4997



$$(-2)\left(x + 2y = 2000\right)$$
$$+\ 2x + y = 3400$$
$$-3y = -600 \Rightarrow y = 200$$
$$x = 1600$$

$$\therefore \text{Total cost} = 1600v_{.05} + 3200v_{.05}^2 + 400v_{.06} + 200v_{.06}^2 = 4981.66$$

9. Alice enters into a four-year interest rate swap. The current 1-year, 2-year, 3-year and 4-year spot rates are 3.5%, 3.5%, 4.0%, and 4.0%, respectively. Determine the swap rate, as a percent and rounded to three decimal places.

A) 3.485%

B) 3.615%

C) 3.765%

D) 3.985%

E) none of the above

Level notional amounts and no deferred period

$$\Rightarrow i = \frac{1 - v_4^4}{v_1 + v_2^2 + v_3^3 + v_4^4} = 0.03985\ldots$$

10. Determine the modified duration of a perpetuity due with level annual payment of K, when calculated using an annual effective interest rate of 5%.

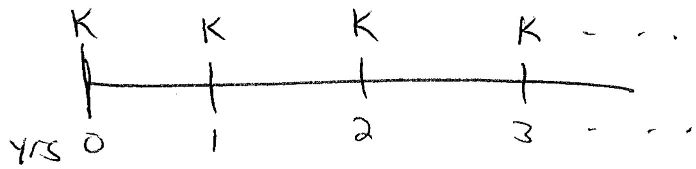
(A) 19.05

(B) 19.65

(C) 20.00

(D) 20.50

(E) 21.00



$$MacD = \frac{Kv + 2Kv^2 + 3Kv^3 + \cdots}{K \cdot \ddot{a}_{\overline{\infty}|}}$$ (K's ~~cancel~~ cancel off)

$$= \frac{v + v^2 + v^3 + \cdots}{\frac{1}{i}(1+i)} = \frac{1/i + 1/i^2}{\frac{1}{i}(1+i)}$$

$$= \frac{\frac{1+i}{i^2}}{\frac{1+i}{i}} = \frac{1}{i} = \frac{1}{.05} = 20$$

$$ModD = v \cdot MacD = 20v = \frac{20}{1.05} = 19.047\ldots$$