MAP 4170 Name:______________________________

Test 4 Date: April 13, 2021

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Tom intends to purchase a car in 5 years. The current cost of the car is 30,000 but Tom expects the cost to increase with inflation at 3% per year. Tom will make deposits at the beginning of each month for the next 5 years in order to have exactly enough money to buy the car. Determine the amount of Tom's monthly deposits → X assuming the deposits earn an interest rate of 8% annual effective.

(A) 470.19

(B) 471.21

(C) 473.32

(D) 473.73

(E) 476.78

In 5 years, Tom's ~~car~~ will cost

$30000(1.03)^5 = 34778.22$

aeir $= .08 \Rightarrow$ m = meir $= (1.08)^{1/12} - 1$

$\therefore X \cdot \ddot{s}_{\overline{60}|m} = 34778.22$

$\Rightarrow X = 473.73$

2. An investment account has a beginning of year balance on January 1 of 100,000. On April 1, there is a deposit, and there are no other transactions for the rest of the year. The account balance immediately before the deposit is 110,000, and the account balance at the end of the year is 126,000. The time-weighted rate of return for the year is 15.5%. Determine the dollar-weighted rate of return for the year.

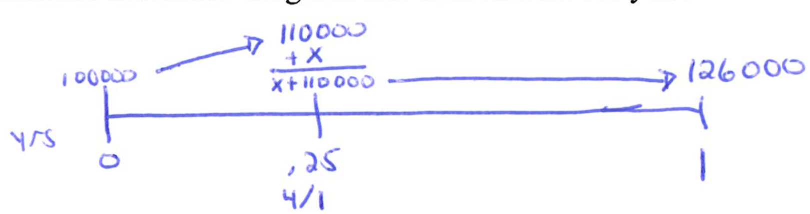
(A) 14.46%

(B) 14.88%

(C) 15.24%

(D) 15.61%

(E) 16.07%



$i_{TW} = 0.155$

$\Rightarrow 1.155 = \frac{110000}{100000} \cdot \frac{126000}{X+110000} \Rightarrow X = 10000$

$i = i_{DW} \Rightarrow 100000(1+i) + 10000(1+.75i) = 126000$

$\Rightarrow i = 0.1488\ldots$

3. The current 1-year spot rate is 3.5%, the current 2-year spot rate is 3.8%, and the current 3-year spot rate is 4.0%. Determine the swap rate for a 3-year interest rate swap.

(A) 3.9%

(B) 4.0%

(C) 4.1%

(D) 4.2%

(E) 4.3%

$$i = \frac{1 - v_3^3}{v_1 + v_2^2 + v_3^3} = \frac{1 - (1.04)^{-3}}{(1.035)^{-1} + (1.038)^{-2} + (1.04)^{-3}}$$

$$\Rightarrow i = 0.03988\ldots$$

4. Jamie takes out a 4-year loan of 100,000 from Bank A with the following provisions:

At the end of each year, Jamie pays 25,000 plus interest on the unpaid balance at the 1-year spot rate in effect at the beginning of the year.

Immediately after taking out the loan, Jamie enters into an interest rate swap on this loan with Bank B in which Jamie pays the fixed swap rate of 3% in all 4 years. After 2 years have passed, the 1-year spot rate is 2.95% and the 2-year spot rate is 3.1%. Determine the market value of the swap from Jamie's position at this time.

(A) – 0.75

(B) 8.25

(C) 17.50

(D) 25.75

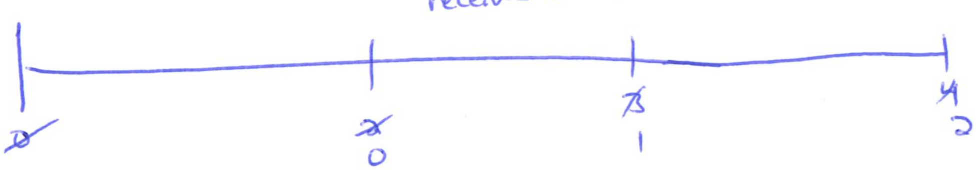
(E) 34.50

$\tilde{S}_1 = .0295 \quad \tilde{S}_2 = .031$

$$\therefore \tilde{f}_{[1,2]} = \frac{(1.031)^2}{1.0295} - 1 = 0.0325\ldots$$

Jamie pays $50000(.03) = 1500$ $\quad 25000(.03) = 750$

receives $50000(.0295) = 1475$ $\quad 25000(.0325\ldots) = 812.55$



$$\therefore MV_2^{Jamie} = (1475 - 1500)\cdot\tilde{v}_1 + (812.55 - 750)\tilde{v}_2$$

$$= \frac{-25}{1.0295} + \frac{62.55}{(1.031)^2} = 34.57$$

5. For a 20-year bond with annual coupons, redeemable at par, the annual coupon rate is equal to the annual yield rate, i. The modified duration of the bond is 14.25. Determine i.

(A) 3.12%

(B) 3.30%

(C) 3.47%

(D) 3.63%

(E) 3.90%

$F = R$ and $r = i$

$\Rightarrow \text{Mod } D = a_{\overline{n}|}$

$\therefore 14.25 = a_{\overline{20}|i}$

$\Rightarrow i = 0.03470\ldots$

6. The price of a bond is 1000 and the modified duration of the bond is 25 when using a 6.0% annual effective yield rate. If the interest rate is changed to 6.2%, annual effective, let C denote the approximate change in the price of the bond using the first order Macaulay approximation, and let D denote the approximate change in the price of the bond using the first order modified approximation. Determine the difference $C - D$.

(A) – 6.52

(B) – 1.27

(C) 0

(D) 1.27

(E) 6.52

$$C = P \cdot \left[\left(\frac{1 + i_{old}}{1 + i_{new}} \right)^{MacD} - 1 \right]$$

$$= 1000 \cdot \left[\left(\frac{1.06}{1.062} \right)^{25(1.06)} - 1 \right] = -48.725\ldots$$

$$D = -P \cdot \text{Mod } D \cdot \Delta i = -1000 \cdot (25) \cdot (.002) = -50$$

$$\therefore C - D = 1.274\ldots$$

7. Liabilities of 10,000 due at the end of 1 year and 35,259 due at the end of 2 years are to be exactly matched using a 1-year zero coupon bond and a 2-year 5% annual coupon bond, redeemable at par. Both bonds can be bought to yield 6% annual effective. Determine the cost to purchase the 1-year zero coupon bond.

(A) 7850

(B) 7925

(C) 8097

(D) 8218

(E) 8321

Assets: at 1: $.05R_2$, R_1; at 2: $1.05R_2$

0 1 2

Liabilities: at 1: 10000; at 2: 35259

$$\left.\begin{aligned} R_1 + .05R_2 &= 10000 \\ 1.05R_2 &= 35259 \end{aligned}\right\} \Rightarrow \begin{aligned} R_2 &= 33580 \\ R_1 &= 8321 \end{aligned}$$

$\therefore$ The cost for the 1-year bond is

$$P_1 = R_1 \cdot v = \frac{8321}{1.06} = 7850$$

8. Liabilities of 10,000 due at the end of 2 year and 25,000 due at the end of 3 years are to be immunized at 3% using 1-year and 4-year zero coupon bonds. Determine the excess of the present value of assets over the present value of liabilities if the interest rate is changed to 4% annual effective.

(A) less than or equal to 1.50

(B) greater than 1.50, but less than or equal to 2.25

(C) greater than 2.25, but less than or equal to 3.00

(D) greater than 3.00, but less than or equal to 3.75

(E) greater than 3.75

A: at 1: R_1; at 4: R_4

0 1 2 3 4

L: at 2: 10000; at 3: 25000

$$(-1)\cdot\left(R_1 v + R_4 v^4 = 10000v^2 + 25000v^3\right) \qquad v = \frac{1}{1.03}$$

$$+\ R_1 v + 4R_4 v^4 = 20000v^2 + 75000v^3$$

$$3R_4 v^4 = 10000v^2 + 50000v^3 \Rightarrow R_4 = 20703$$

$$\Rightarrow R_1 = 14327.46$$

At 4%, $E = \left(\frac{14327.46}{1.04} + \frac{20703}{1.04^4}\right) - \left(\frac{10000}{1.04^2} + \frac{25000}{1.04^3}\right) = 2.94$

9. Consistent with the current term structure of interest rates, a 5-year 1000 face-value bond with 4% annual coupons and a redemption value of 1200 sells for 168.40 more than a 5-year 1000 face-value bond, redeemable at par, with 4% annual coupons. Determine the 5-year spot rate.

(A) 3.1%

(B) 3.2%

(C) 3.3%

(D) 3.4%

(E) 3.5%

For the bond redeemable at par, we have

$$P = 40v_1 + 40v_2^2 + 40v_3^3 + 40v_4^4 + 1040v_5^5$$

For the other bond, we have

$$P + 168.40 = 40v_1 + 40v_2^2 + 40v_3^3 + 40v_4^4 + 1240v_5^5$$

∴ subtracting the first equation from the second, we get

$$168.40 = 200v_5^5$$

$$\Rightarrow (1+s_5)^5 = \frac{200}{168.4} \Rightarrow s_5 = 0.03499\ldots$$

10. A payment of 2000 is due at the end of 2 years, and another payment of 5000 is due at the end of 5 years. Using an annual effective interest rate of 5%, denote the Macaulay convexity of this set of payments by C and the modified convexity of this set of payments by D. Determine the ratio C/D.

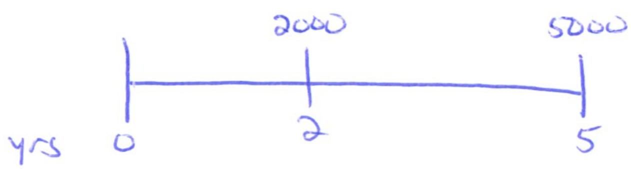
(A) 0.903

(B) 0.948

(C) 1.000

(D) 1.054

(E) 1.107



$$v = \frac{1}{1.05}$$

$$C = \frac{4 \cdot 2000v^2 + 25 \cdot 5000v^5}{2000v^2 + 5000v^5} = 18.35357\ldots$$

D: $P(i) = 2000(1+i)^{-2} + 5000(1+i)^{-5}$

$P'(i) = -4000(1+i)^{-3} - 25000(1+i)^{-6}$

$P''(i) = 12000(1+i)^{-4} + 150000(1+i)^{-7}$

$$\therefore D = \frac{12000v^4 + 150000v^7}{2000v^2 + 5000v^5} = 20.3211\ldots$$

$$\therefore C/D = 0.903\ldots$$