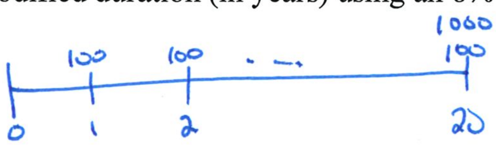
MAP 4170 Name:_________________________

Test 4 Date: April 19, 2022

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A bond will pay a coupon of 100 at the end of each year for the next twenty years and will pay the face value of 1000 at the end of the twenty-year period.

   Calculate the bond's modified duration (in years) using an 8% annual effective interest rate.



   (A) 8.73

   (B) 9.43

   (C) 10.18

   (D) 11.00

   (E) 11.88

$$MacD = \frac{100(Ia)_{\overline{20|}.08} + 20\cdot 1000 v^{20}}{100\, a_{\overline{20|}.08} + 1000\, v^{20}} = 10.18\ldots$$

$$ModD = MacD \cdot v = \frac{10.18\ldots}{1.08} = 9.428\ldots$$

2. You are given the following information on a portfolio consisting of two bonds:

   Bond A is a 1000 face-value 10-year bond. Using an annual effective interest rate of 9%, it has a price of 1000 and a Macaulay duration of 7 years.

   Bond B is a 10-year zero coupon bond with redemption value of 1000.

   In what range (in years) is the Macaulay duration, D, of the portfolio using an annual effective interest rate of 9%?

   (A) D < 7.8

   (B) 7.8 < D < 8.0

   (C) 8.0 < D < 8.2

   (D) 8.2 < D < 8.4

   (E) D > 8.4

$$P_A = 1000 \qquad P_B = 1000\, v_{.09}^{10} = 422.41$$

$$\therefore W_A = \frac{1000}{1422.41} = 0.703$$

$$W_B = \frac{422.41}{1422.41} = 0.297$$

$$MacD_A = 7 \qquad MacD_B = 10$$

$$\therefore MacD_{Portfolio} = 0.703(7) + 0.297(10) = 7.89$$

3. The price of a bond is 1000 at a 6.0% annual effective interest rate. If the interest rate is changed to 6.2% annual effective, the first order Macaulay approximation yields a price of 964.67. Determine the modified duration of the bond at a 6% annual effective interest rate.

$i_{old} = .06$

$i_{new} = .062$

1st order Macaulay approximation:

$$\Delta P = P\left[\left(\frac{1+i_{old}}{1+i_{new}}\right)^{MacD} - 1\right]$$

(A) 16

(B) 17

(C) 18

(D) 19

(E) 20

$$\Delta P = 964.67 - 1000 = -35.33$$

$$\therefore -35.33 = 1000\left[\left(\frac{1.06}{1.062}\right)^{MacD} - 1\right]$$

$$\Rightarrow MacD = 19.08\cdots$$

$$\Rightarrow ModD = MacD \cdot v = \frac{19.08\cdots}{1.06} = 18.0$$

4. On January 1, 2021, an investment fund had a balance of 100,000. On May 1, 2021, a withdrawal of 3000 was made. The balance in the fund immediately before the withdrawal was 103,000. On September 1, 2021, a deposit of 6000 was made. The balance in the fund immediately before the deposit was 106,000. There were no other deposits or withdrawals during 2021, and the 2021 calendar year time weighted return was 9.18%. Determine the 2021 calendar year dollar weighted return.

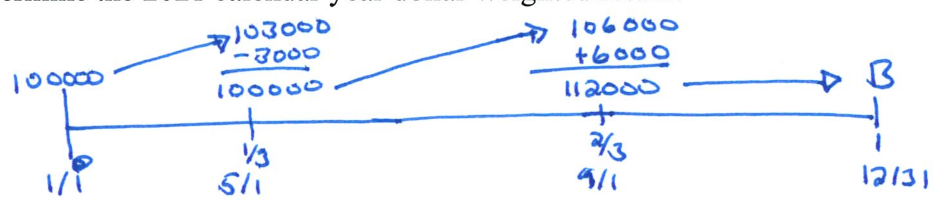
(A) 6%

(B) 7%

(C) 8%

(D) 9%

(E) 10%



$$i_{TW} = .0918 \Rightarrow 1.0918 = \frac{103000}{100000} \cdot \frac{106000}{100000} \cdot \frac{B}{112000}$$

$$\Rightarrow B = 112000$$

$i = i_{DW}$:

$$100000(1+i) - 3000\left(1+\tfrac{2}{3}i\right) + 6000\left(1+\tfrac{1}{3}i\right) = 112000$$

$$\Rightarrow i = .09$$

5. Catherine wants to have 1.5 million "in today's dollars" in her retirement account at the end of thirty years. She will make level deposits at the beginning of each month for the thirty-year period in order to accomplish her goal. She assumes that she can get a 9% compounded monthly interest rate on her deposits, and she assumes inflation over the next thirty years will be 2.4% compounded monthly. Adam and Brianna are each asked to determine the amount of the monthly deposits.

   Adam determines the payments by setting the accumulated value of the payments, using the real rate of return, equal to 1.5 million. Brianna determines the payments by first determining the accumulated value of the payments at the nominal rate. She then discounts this accumulated value, using the inflation rate, for 30 years and sets the result equal to 1.5 million. Letting A denote Adam's answer and B denote Brianna's answer, determine $B - A$.

   all Meir's $\left\{ \begin{array}{l} i = \frac{.09}{12} = .0075 \qquad r = \frac{.024}{12} = .002 \\ \therefore\ i' = \frac{1.0075}{1.002} - 1 = .005489\ldots \end{array} \right.$ (real rate of return)

   (A) 345

   (B) 350

   (C) 355

   (D) 360

   (E) 365

   $A:\ C \cdot \ddot{s}_{\overline{360}|i'} = 1500000 \Rightarrow C = 1326 = A$

   $B:\ \left(C \cdot \ddot{s}_{\overline{360}|.0075}\right) \cdot v_{.002}^{360} = 1500000$

   $\Rightarrow C \cdot \ddot{s}_{\overline{360}|.0075} = 1500000(1.002)^{360} \Rightarrow C = 1670 = B$

   $\therefore B - A = 344$

6. The current 1-year spot rate is 3.0% and the current 3-year spot rate is 3.4%. Determine the current 2-year spot rate that would be consistent with a 3-year swap rate of 3.387% on level notional amount interest rate swaps.

   (A) 3.0%

   (B) 3.1%

   (C) 3.2%

   (D) 3.3%

   (E) 3.4%

   $i = \frac{1 - v_3^3}{v_1 + v_2^2 + v_3^3}$

   $\Rightarrow .03387 = \frac{1 - (1.034)^{-3}}{(1.03)^{-1} + (1+s_2)^{-2} + (1.034)^{-3}}$

   $\Rightarrow s_2 = .0301\ldots$

7. Liabilities of 9621 due at the end of 1 year and 13860 due at the end of 2 years are to be exactly matched using a 1-year 3% annual coupon bond and a 2-year 5% annual coupon bond. Both bonds have a face value of 1000 and are redeemable at par. Determine the number of the 1-year bonds needed.

m = # of 1-year bonds
n = # of 2-year bonds

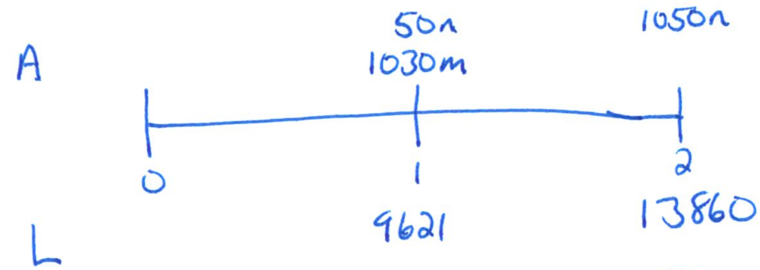
(A) 7.9

(B) 8.1

(C) 8.3

(D) 8.5

[✓ (E) 8.7]



$$\left.\begin{array}{l} 50n + 1030m = 9621 \\ 1050n = 13860 \end{array}\right\} \Rightarrow n = 13.2$$

$\boxed{m \doteq 8.7}$

8. Liability payments of 1000 due at the end of 5 years and another 1000 due at the end of 10 years are to be offset with asset payments of X that will be paid in 3 years and Y that will be paid in 8 years in such a way that the present value and Macaulay duration of the assets are equal to the present value and Macaulay duration of the liabilities, respectively, using an annual effective discount rate of 10%. Determine X.

aedr = .1
$\Rightarrow$ adf = .9 = v

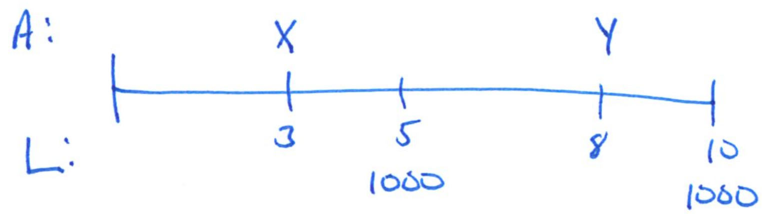
(A) 155

(B) 215

(C) 220

(D) 290

[✓ (E) 295]



PV: $\left(Xv^3 + Yv^8 = 1000v^5 + 1000v^{10}\right)(8)$

numerator of MacD: $3Xv^3 + 8Yv^8 = 5000v^5 + 10000v^{10}$

$-$

$5Xv^3 = 3000v^5 - 2000v^{10}$

$\therefore X = 294.68$

9. A two-year bond with annual coupons of 100 and redemption value of 1000 is priced at 1,045.34. The one-year forward rate for year two is 8%. Determine the current $\underbrace{\text{one-year spot rate}}_{S_1}$ that is consistent with the pricing of the bond.

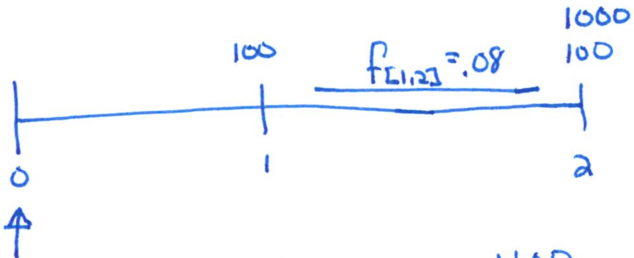
   (A) 4%

   (B) 5%

   (C) 6%

   (D) 7%

   (E) 8%



$$1045.34 = \frac{100}{1+S_1} + \frac{1100}{(1.08)(1+S_1)}$$

$$\Rightarrow S_1 = .07$$

10. A payment of 5000 is due at the end of 2 years, and another payment of 2000 is due at the end of 5 years. Using an annual effective discount rate of 10%, determine the Macaulay convexity of this set of payments.

    aedr = .1

    $\Rightarrow v = .9 = \text{adf}$

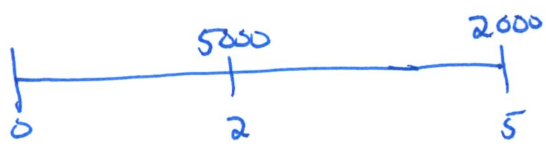
    (A) less than 8.6

    (B) greater than or equal to 8.6, but less than 8.7

    (C) greater than or equal to 8.7, but less than 8.8

    (D) greater than or equal to 8.8, but less than 8.9

    (E) greater than or equal to 8.9



$$\text{Mac } C = \frac{2^2 \cdot 5000 v^2 + 5^2 \cdot 2000 v^5}{5000 v^2 + 2000 v^5} = 8.74$$