

Elementary Loss Models

In this section, X denotes a non-negative random variable representing loss amount.

Conditional vs Unconditional pdfs

$$f_{X|X>d}(x | x > d) = \left\{ \begin{array}{l} 0 \dots (X \leq d) \\ \frac{f_X(x)}{\Pr(X > d)} \dots (X > d) \end{array} \right\}$$

Summing n independent and identically distributed random variables ($X_i \sim X$)

$$\begin{aligned} S &= \sum_{i=1}^n X_i \\ E[S] &= n \cdot E[X] \\ \text{Var}(S) &= n \cdot \text{Var}(X) \end{aligned}$$

Deductibles – Y_L denotes the random variable representing the amount paid per *loss* by an insurer after a deductible of d is applied.

$$\begin{aligned} Y_L = (X - d)_+ &= \left\{ \begin{array}{l} 0 \dots (X \leq d) \\ X - d \dots (X > d) \end{array} \right\} \\ E[(Y_L)^n] &= E[(X - d)^n | X > d] \cdot \Pr(X > d) = \int_d^{\infty} (x - d)^n \cdot f_X(x) dx \end{aligned}$$

Deductibles – Y_p denotes the random variable representing the amount paid per *payment* by an insurer after a deductible of d is applied.

$$Y_p = Y_L | Y_L > 0 = X - d | X > d$$

$$E[(Y_p)^n] = E[(Y_L)^n | Y_L > 0] = \frac{E[(Y_L)^n]}{\Pr(X > d)} = E[(X - d)^n | X > d] = \frac{\int_d^{\infty} (x - d)^n \cdot f_X(x) dx}{\Pr(X > d)}$$

Policy Limits – Y denotes the random variable representing the amount paid per loss by an insurer after a policy limit of L is applied.

$$Y = X \wedge L = \begin{cases} X \cdots (X \leq L) \\ L \cdots (X > L) \end{cases}$$

$$E[Y^n] = E[X^n | X \leq L] \cdot \Pr(X \leq L) + L^n \cdot \Pr(X > L) = \int_0^L x^n \cdot f_X(x) dx + L^n \cdot \int_L^\infty f_X(x) dx$$

Proportional Insurance – Y denotes the random variable representing the amount paid per loss by an insurer that insures a proportion α of the loss ($0 < \alpha < 1$).

$$Y = \alpha \cdot X$$

$$E[Y^n] = \alpha^n \cdot E[X^n]$$

$$\text{Var}(Y) = \alpha^2 \cdot \text{Var}(X)$$

Often Tested Facts

1. $X \sim U[0, c] \Rightarrow X - d | X > d \sim U[0, c - d]$
2. $X \sim EX(\text{mean} = \mu) \Rightarrow X - d | X > d \sim EX(\text{mean} = \mu)$
(memoryless property of exponential distribution)
3. $X = (X \wedge c) + (X - c)_+$