Elementary Loss Models

In this section, $X$ denotes a non-negative random variable representing loss amount.

Conditional vs Unconditional pdfs

$$f_{X|X>d}(x \mid x > d) = \begin{cases} 
0 \cdot (X \leq d) \\
\frac{f_X(x)}{\Pr(X > d)} \cdot (X > d)
\end{cases}$$

Summing $n$ independent and identically distributed random variables ($X_i \sim X$)

$$S = \sum_{i=1}^{n} X_i$$

$$E[S] = n \cdot E[X]$$

$$\text{Var}(S) = n \cdot \text{Var}(X)$$

Deductibles – $Y_L$ denotes the random variable representing the amount paid per loss by an insurer after a deductible of $d$ is applied.

$$Y_L = (X - d)_+ = \begin{cases} 
0 \cdot (X \leq d) \\
X - d \cdot (X > d)
\end{cases}$$

$$E[(Y_L)^n] = E[(X - d)^n \mid X > d] \cdot \Pr(X > d) = \int_{d}^{\infty} (x - d)^n \cdot f_X(x) dx$$

Deductibles – $Y_P$ denotes the random variable representing the amount paid per payment by an insurer after a deductible of $d$ is applied.

$$Y_P = Y_L \mid Y_L > 0 = X - d \mid X > d$$

$$E[(Y_P)^n] = E[(Y_L)^n \mid Y_L > 0] = \frac{E[(Y_L)^n]}{\Pr(X > d)} = E[(X - d)^n \mid X > d] = \frac{\int_{d}^{\infty} (x - d)^n \cdot f_X(x) dx}{\Pr(X > d)}$$
Policy Limits – $Y$ denotes the random variable representing the amount paid per loss by an insurer after a policy limit of $L$ is applied.

\[ Y = X \land L = \begin{cases} X \cdot (X \leq L) \\ L \cdot (X > L) \end{cases} \]

\[ E[Y^n] = E[X^n | X \leq L] \cdot \Pr(X \leq L) + L^n \cdot \Pr(X > L) = \int_0^L x^n \cdot f_X(x)dx + L^n \cdot \int_L^\infty f_X(x)dx \]

Proportional Insurance – $Y$ denotes the random variable representing the amount paid per loss by an insurer that insures a proportion $\alpha$ of the loss ($0 < \alpha < 1$).

\[ Y = \alpha \cdot X \]

\[ E[Y^n] = \alpha^n \cdot E[X^n] \]

\[ \text{Var}(Y) = \alpha^2 \cdot \text{Var}(X) \]

Often Tested Facts

1. $X \sim U[0, c] \Rightarrow X - d \mid X > d \sim U[0, c - d]$  
2. $X \sim EX(\text{mean} = \mu) \Rightarrow X - d \mid X > d \sim EX(\text{mean} = \mu)$  
   (memoryless property of exponential distribution)
3. $X = (X \land c) + (X - c)_+$