Print your name:

Test 1 Review
Matrix algebra

Due: February 19\textsuperscript{th}

February 11, 2013

This is Test 1 review. Remember that no calculators will be allowed during Test 1. These questions serve as a preparation. You may use SCILAB to check your results, but your approach to solving the questions below should be stated clearly. Make sure to show me your work and explain the steps that lead to your answer. State any theorem you use explicitly.

Question 1 Find the reduced row echelon form of the following matrix:

\[ A = \begin{bmatrix}
2 & -3 & -1 & 0 \\
3 & 2 & 2 & 2 \\
1 & 5 & 3 & 2 \\
\end{bmatrix} \]

Question 2 Find \( EA - 2B \) if \( A = \begin{pmatrix}
1 & 0 & -4 \\
-1 & 2 & 3 \\
\end{pmatrix}, \quad B = \begin{pmatrix}
2 & 1 & -1 \\
-2 & 0 & -3 \\
\end{pmatrix} \) and \( E = \begin{pmatrix}
-2 & 6 \\
4 & 2 \\
\end{pmatrix} \)

- \( \begin{pmatrix}
-12 & 10 & 28 \\
6 & 4 & -4 \\
\end{pmatrix} \)
- \( \begin{pmatrix}
-4 & 14 & 24 \\
-2 & 4 & -16 \\
\end{pmatrix} \)
- \( \begin{pmatrix}
-12 & 10 & 24 \\
-2 & 4 & -4 \\
\end{pmatrix} \)
- None of these

Question 3 How many solutions does the system below have?

\[
\begin{bmatrix}
2 & -2 & 1 \\
2 & 1 & 2 \\
4 & 2 & 4 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix}
=
\begin{bmatrix}
2 \\
-1 \\
1 \\
\end{bmatrix}
\]

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Question 4 Consider the following matrix in reduced row echelon form:

\[
M = \begin{bmatrix}
1 & 0 & 5/6 & 0 \\
0 & 1 & 1/3 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

What is the rank of M? Does the corresponding system have solution(s)? If so, find it (them).

Question 5 Let

\[
A = \begin{bmatrix}
3 & -9/2 \\
2 & -3
\end{bmatrix}
\]

Is A nilpotent?

Question 6 Find the determinant of the following matrix using the method of your choice.

\[
M = \begin{bmatrix}
1 & 0 & 5 \\
0 & 1 & 2 \\
1 & 0 & 1
\end{bmatrix}
\]

Question 7 Find the determinant of the following matrix using the method of your choice.

\[
B = \begin{bmatrix}
1 & 3 & 2 \\
3 & 1 & 2 \\
4 & 12 & 8
\end{bmatrix}
\]
Question 8  In the two previous questions, are $M$ and $B$ invertible? If so, find their inverse.

Question 9  Does the following system have a solution?
\[
\begin{align*}
    x - 2y + 3z &= 7 \\
    2x + y + z &= 4 \\
    -3x + 2y - 2z &= -10
\end{align*}
\]

Question 10  Consider the vectors
\[
X_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 2 \\ 5 \\ 5 \end{pmatrix}, \quad X_3 = \begin{pmatrix} 2 \\ 5 \\ 2 \end{pmatrix}.
\]

Is the vector $W = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$ a linear combination of $(X_1, X_2, X_3)$?