## 1. Section 13.3 Geometric Sequences

**Example 1.1.** *What do you notice about the following sequences:*

*(1)* $\{2, 5, 8, 11, 14, \dots\}$

*(2)* $\{2, 6, 18, 54, 162, \dots\}$

**Definition 1.1.** *A* **geometric sequence** *is a sequence where the "next" term is*

_____

*More precisely,* $\{a_n\}$ *is geometric if*

$a_{n+1} =$ _____

*where* $r$ *is a constant.*

*(1)* $a_1 = a$ *is the* **initial** *term.*

*(2)* $r$ *is called the* _____

*(3)* $r =$ _____

**Example 1.2.** *Find the common ratio in each of the geometric sequence in example 1.1*

**Example 1.3.** *Which sequences are geometric?*

*(1)* $\{1, 3, 9, 27, \dots\}$

*(2)* $\{1, 3, 5, 7, 9 \dots\}$

*(3)* $\{3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8} \dots\}$

## 2. Finding the $n$-th term

**Theorem 2.1** (Formula)**.** *Assume* $\{a_n\}$ *is a geometric sequence with common ratio* $r$*. Then*

$a_n =$ _____

**Example 2.1.** *Given the first term of a geometric sequence is* $a_1 = 4$ *and the common ratio is* $r = -\frac{2}{3}$*, find the term below*

*(1)* $a_4$

*(2) the 30-th term.*

*(3) the n-th term.*

**Example 2.2.** *Find the n-th term of a geometric sequence given the first term is* $a_1 = 4$ *and the common ratio is* $r = \sqrt[3]{2}$

**Example 2.3.** *Suppose* $\{a_n\}$ *is a geometric sequence with* $a_{16} = 15$ *and* $a_{20} = 30$.

*(1) Find* $a_1$.

*(2) Find the common ratio.*

*(3) Find* $a_{10}$.

## 3. Finding Finite Sums

**Notation 3.1** (Notation)**.** $S_n =$ _____

**Theorem 3.1** (Formula)**.** $S_n =$ _____

**Example 3.1.** *Find* $2 + \frac{2}{5} + \frac{2}{5^2} + \cdots + \frac{2}{5^{31}}$.

**Example 3.2.** *Find* $2 + \frac{2}{5} + \frac{2}{5^2} + \cdots + \frac{2}{5^{n-1}}$.

## 4. Geometric Series

**Definition 4.1.** *A* **geometric series** *is an infinite sum of a geometric sequence.*

**Notation 4.1.**

$$S = \sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

**Theorem 4.1** (Formula)**.** $S = \sum_{k=1}^{\infty} ar^{k-1} =$ _____

*if* $|r| < 1$*. When* $|r| \geq 1$ *then the sum is* _____

**Example 4.1.** *Find* $2 + \frac{2}{5} + \frac{2}{5^2} + \cdots + \frac{2}{5^{n-1}} + \cdots$.

**Example 4.2.** *Determine if the sum converges or diverges. If it converges, find the sum.*

$$1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \cdots$$

**Example 4.3.** *Determine if the sum converges or diverges. If it converges, find the sum.*

$$\sum_{k=1}^{\infty} \frac{2^{k-1}}{5}$$

## 5. Mixed Examples

**Example 5.1.** *Determine if the sequence is arithmetic, geometric, or neither. If it is arithmetic or geometric find the sum of the first 50 terms.*

*(1)* $\left\{\dfrac{n}{n+1}\right\}$

*(2)* $\{3-5n\}$

*(3)* $\{100^{-n/2}\}$