1. **SECTION 5.4 SOLVING POLYNOMIAL AND RATIONAL INEQUALITIES**

**The Sign Chart Method (usually taught in College Algebra)**

Step 1. Add or subtract so that all terms of the inequality are on one side and 0 is on the other side. You need to get the equation in one of the following forms:

\[
\begin{align*}
    f(x) &> 0 \\
    f(x) &\geq 0 \\
    f(x) &< 0 \\
    f(x) &\leq 0
\end{align*}
\]

Step 2. Simplify the expression \( f(x) \). Combine and factor.

Step 3. Find where \( f(x) \) is equal to 0 or undefined (i.e. where is the numerator and where is the denominator equal to zero). We will call these values the **Partitioning Values**.

Step 4. Put these numbers on a number line. The Partition values will separate (partition) the number line into one or more intervals.

Step 5. Make a chart by making each interval in Step 4 a column and each factor in \( f(x) \) a row. You may combine more than one factor in the rows, but every factor should be accounted for.

Step 6. Select a test number in each interval and determine if the factor in each row is positive or negative.

Step 7. Add a row in your table for \( f(x) \) and determine the sign of \( f(x) \) in each interval (column) by counting the negatives in each column (even number of negatives makes \( f \) positive and an odd number of negatives makes \( f \) negative).

Step 8. Answer the question based on which of the inequalities

\[
\begin{align*}
    f(x) &> 0 \\
    f(x) &\geq 0 \\
    f(x) &< 0 \\
    f(x) &\leq 0
\end{align*}
\]

you are to solve and the signs given for \( f \) in your table.

**Remark 1.1.** *This method works because a polynomial or rational function can only change signs where it equals zero or is undefined.*
Example 1.1. Solve $\frac{1}{3-x} \geq 2$.

Example 1.2. Solve $\frac{4x - 1}{(x + 2)(x + 3)} \leq 0$.

(A) $[-3, -2] \cup [1/4, \infty)$
(B) $(-3, -2) \cup [1/4, \infty)$
(C) $(-\infty, -3] \cup [-2, 1/4]$ (D) $(-\infty, -3) \cup (-2, 1/4]$

Remark 1.2. The union symbol in sets is not the same as the letter $U$ and does not appear on your keyboard. To enter the union in MML you have to use the math tool palette.
2. New Tools in Precalculus

You have learned tools to quickly sketch a variety of polynomials and rational functions. Sketch the graph of the function and then determine solution to the inequality using the graph.

Example 2.1. Solve \( x^2 - x - 6 > 0 \) by sketching the graph.

We can also use a combination of the sign chart and information about the graphs of functions that we have learned this semester to make the solution quicker to find.

Example 2.2. Solve \( -2(x + 4)^2(x^2 + 9)(4 - x)^3 > 0 \).
Example 2.3. Solve \( \frac{x^4(x^2 + 4)(x^2 - 4)}{(2x + 1)^2(x - 4)^3} \leq 0. \)