1. Chapter 1 Section 9: Using Simplification to Evaluate Limits Algebraically

In this section, we further develop the idea of finding a limit by using algebra to re-write the expression. We can then use the following property to evaluate many limits.

Recall from section 1.8 notes: If $f(x) = g(x)$ for all x in an open interval containing a (except possibly at a), then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$.

Example 1.1 (1.9 WP Homework Question 3, Text 19)**.** *Use algebra to evaluate the limit.* $\lim_{h \to 0} \frac{1/(4+h) - 1/4}{h}$.

Example 1.2. *Use algebra to evaluate the limit.* $\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5}$

Example 1.3 (1.9 practice question in text 43)**.** *Find all values for the constant k such that the limit exists.*

$$\lim_{x \to 5} \frac{x^2 - kx + 5}{x^2 - 2x - 15}$$

Example 1.4 (1.9 WP Homework Question 8, Text 57)**.** *Evaluate the limit using algebra.*

$$\lim_{x \to \infty} \frac{2e^{3x} - 1}{5e^{3x} + e^x + 1} =$$

2. The Squeeze Theorem

Theorem 2.1. *If $f, a,$ and b are functions such that*

$$b(x) \leq f(x) \leq a(x)$$

for all x around c (except possibly at c), then

$$\lim_{x\to c} b(x) \leq \lim_{x\to c} f(x) \leq \lim_{x\to c} a(x)$$

Theorem 2.2 (The Squeeze Theorem)**.** *If $f, a,$ and b are functions such that*

$$b(x) \leq f(x) \leq a(x) \text{ and } \lim_{x\to c} b(x) = L = \lim_{x\to c} a(x)$$

for all x around c (except possibly at c), then

$$\lim_{x\to c} f(x) = L$$

Example 2.1 (1.9 Text 60)**.** *Prove:* $\displaystyle\lim_{x\to\infty} \frac{\sin^2 x}{4x^3 + 5x + 1} = 0$

Example 2.2 (1.9 Text 38)**.** *Find* $\lim_{x\to 0} f(x)$ *if, for all* x*,*

$$4\cos(2x) \le f(x) \le 3x^2 + 4.$$