Chapter 2 Section 1: How do we Measure Speed?

1. Velocity

Definition 1.1. *The* **average velocity** *of an object over a given interval of time, $[t_1, t_2]$, is the change in the position divided by the change in time.*

$$v_{ave} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Definition 1.2. *The* **instantaneous velocity** *of an object at a given time, $t = a$ is the limiting value of the average velocity over the time interval between t and a as t approaches a. The* **speed** *of an object is the magnitude, or absolute value of velocity. Using limit notation this is stated*

$v(a) =$

Example 1.1 (2.1 WP Homework Question 4, Text 11)**.** *If a rock is thrown upward on the planet Mars with an initial velocity of 10 m/s, its approximate height in meters t seconds later is given by $s(t) = 10t - 1.86t^2$.*

(1) Find the average velocity over the given time intervals:

(i) $[1, 2]$ *(ii)* $[1, 1.5]$ *(iii)* $[1, 1.1]$
(iv) $[1, 1.01]$ *(v)* $[1, 1.001]$

Solution

	t	$10t - 1.86t^2$	v_{ave} *over* $[1, t]$
(i)	2	12.56	4.42
(ii)	1.5	10.815	5.35
(iii)	1.1	8.7494	6.094
(iv)	1.01	8.202614	6.2614
(vi)	1.001	8.14627814	6.27814

(2) Estimate the instantaneous velocity when $t = 1$ based on the table of values.

Example 1.2. *Recall from example 1.1 that if a rock is thrown upward on the planet Mars with an initial velocity of 10 m/s, its approximate height in meters t seconds later is given by $s(t) = 10t - 1.86t^2$.*

(1) Express the value of the instantaneous velocity when $t = 1$ as a limit.

(2) (2.1 WP Homework Question 7, 8; Text 35, 38) *Use algebra to find the limit in part 1.*

2. Visualizing Velocity

Definition 2.1. *(Informal) The line* **tangent** *to the graph of a function at a point is the line that "best approximates" the graph at that point. The* **slope** *of a curve at a point is the slope of the tangent line at that point.*

We use the previous example to explore how to visualize velocity.

Example 2.1. *Desmos: https://www.desmos.com/calculator/ncdh4uqmz5*
Recall from example, 1.1, that if a rock is thrown upward on the planet Mars with an initial velocity of 10 m/s, its approximate height in meters t seconds later is given by $s(t) = 10t - 1.86t^2$.

(1) Sketch a graph of s in a coordinate plane with the horizontal axis represented by t and the vertical axis represented by s.
(2) On your sketch draw the secant line through $(1, s(1))$ and $(2, s(2))$.
(3) On your sketch draw the tangent line at $(1, s(1))$. That is, draw the line that best approximate the graph of the function at the given point.
(4) Identify from the previous examples the slopes of the lines in parts 2 and 3

Example 2.2 (2.1 WP Homework Question 6; Text 27)**.** *The graph of a function $y = f(x)$ is given below. Determine the signs of the slopes at each labeled point, then put the slopes in increasing order.*