Chapter 2 Section 2: The Derivative at a Point

1. Average Rate of Change

(1) The average velocity of a particle with position at time t given by $s(t)$ over the time interval $[a, b]$ is

$v_{ave} =$

(2) The slope of the secant line of $y = f(x)$ through $(a, f(a))$ and $(b, f(b))$ is

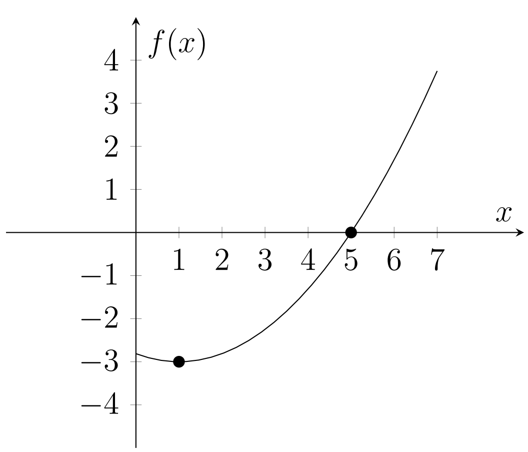
$m_{sec} =$

Remark 1.1. *A line secant to a curve is simply a line through two distinct points on the curve. The word secant here has nothing to do with the word secant in trigonometry. The context should make it clear what is meant when you see the word "secant."*

Notice the common structure of the above formulas. This structure occurs in a wide variety of application, so we define this structure more generally.

Definition 1.1. *The* **average rate of change** *of $y = f(x)$ from the point $(a, f(a))$ to $(b, f(b))$ (or over the interval $[a, b]$) is $\dfrac{f(b) - f(a)}{b - a}$. If $b = a + h$, then this expression becomes $\dfrac{f(a+h) - f(a)}{h}$.*

Example 1.1 (2.2 Text 22)**.** *The graph of $y = f(x)$ is given. Sketch a representation of each quantity on the graph and determine the values.*

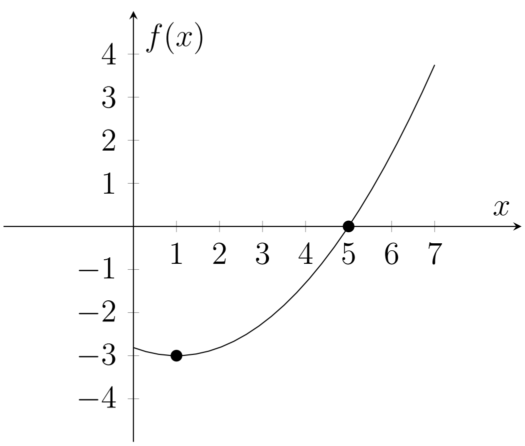


(1) $f(5)$ *(2)* $f(5) - f(1)$ *(3)* $\dfrac{f(5) - f(1)}{5 - 1}$

2. Instantaneous Rate of Change: The Derivative

Definition 2.1. *The* **(instantaneous) rate of change** *or the* **derivative** *of* $y = f(x)$ *at* $(a, f(a))$ *is* $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$. *Equivalently,* $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$. *If this limit exists, then we say* f *is* **differentiable** *at* a.

Example 2.1 (2.2 Text 15, 22). *The graph of* $y = f(x)$ *is given. Sketch a representation the quantitys on the graph and approximate the values.*



(1) $f'(5)$

(2) $f'(1)$

Remarks 2.1.

(1) The derivative at a point is the rate of change at that point. It may be used to approximate the change in function value for small changes in the independent variable.

(2) Here are some of the common ways to write the derivative of f *at* a.

$$f'(a) = y'(a) = \frac{d}{dx}f(x)\Big|_{x=a} = \frac{dy}{dx}\Big|_{x=a} = Df(a) = D_x f(a) =$$

Example 2.2 (2.2 WP Homework Questions 8, Text 59)**.** *Let* $f(x) = x^2 + 2x$.

1. *(1) Sketch the graph of the function and the tangent line at* $x = 1$. *(Desmos: https://www.desmos.com/calculator/dtm2yjikbo)*
2. *(2)* (2.2 WP Homework Questions 8, Text 59) *Find* $f'(1)$ *by estimating using the graph and then algebraically using the definition of the derivative (difference quotients).*
3. *(3)* (2.2 WP Homework Questions 10, Text 66) *Find the equation of the line tangent to the graph of* $f(x) = x^2 + 2x$ *at* $x = 1$.
4. *(4)* (2.2 WP Homework Questions 3, Text 21) *Use part 3 to estimate the value of* $f(1.1)$.

Example 2.3. *The atmospheric pressure, P, at a point h kilometers above the surface of the earth is $P = f(h) = 760e^{-h/7}$ torr. Find $f(1)$ and estimate $f'(1)$. What do your answers tell you about the pressure?*

(1) (2.2 Text 39) *Use the following table of values to solve this problem*

h	0.7	0.8	0.9	1	1.1	1.2
$f(h)$	687.68	677.92	668.31	658.83	649.48	640.27

(2) (2.2 WP Homework Questions 7, Text 40) *Now use this table of values to solve this problem. Pay attention to the headings in each table.*

h	$\frac{f(1+h)-f(1)}{h}$	h	$\frac{f(1+h)-f(1)}{h}$
0.1	-93.44908921	-0.1	-94.79365739
0.01	-94.05097671	-0.01	-94.18543126
0.001	-94.11144957	-0.001	-94.12489502
0.0001	-94.1174997	-0.0001	-94.11884425
0.00001	-94.11810475	-0.00001	-94.1182392
0.000001	-94.1181652	-0.000001	-94.1181787
0.0000001	-94.11817086	-0.0000001	-94.11817254
0.00000001	-94.11816748	-0.00000001	-94.11817592