Chapter 2 Section 3: The Derivative Function

1. Recall the derivative at a point

Definition 1.1. *The* **(instantaneous) rate of change** *or the* **derivative** *of* $y = f(x)$ *at* $(a, f(a))$ *is* $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$. *Equivalently,* $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

Example 1.1. *Let* $f(x) = \frac{1}{x+3}$. *Use desmos or another graphing utility to approximate* $f'(-2)$, $f'(-1)$, $f'(0)$, $f'(1)$, *and* $f'(2)$. *Plot the points* $(a, f'(a))$.

(https://www.desmos.com/calculator/0yris6fm6y)

2. Definition of the Derivative

The definition of the derivative at a point may be extended to define the derivative as a function whose domain is all the real numbers for which the original function is differentiable.

Definition 2.1. *The* **derivative** *(function) of* $y = f(x)$ *is* $f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$. *Equivalently,* $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Notations: $f'(x) = y' = \dot{y} = \frac{d}{dx} f(x) = \frac{dy}{dx} = Df(x) = D_x f(x) =$

Example 2.1 (2.3 WP Homework Questions 3, Text 23)**.** *Let* $f(x) = \dfrac{1}{x+3}$*. Find* $f'(x)$ *using the definition of the derivative (the difference quotient). Find the domain of* f *and the domain of* f'*. Sketch the graph of* $y = f'(x)$ *using a graphing utility.*

(https://www.desmos.com/calculator/kcy8rk8mvh)

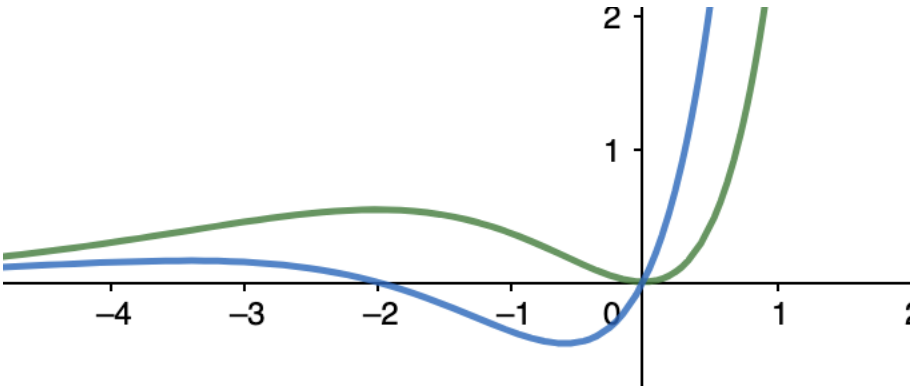
Remarks 2.1.

(1) If f' *is positive on the interval* (a, b)*, then* f *is increasing on that interval.*
(2) If f' *is negative on the interval* (a, b)*, then* f *is decreasing on that interval.*
(3) If f' *is zero on the interval* (a, b)*, then* f *is constant on that interval.*
(4) The domain of f' *must be a subset of the domain of* f*, (equal or a proper subset).* f *is differentiable on the domain of* f'*.*

Example 2.2 (2.3 WP Homework Questions 4, Text 33)**.** *Draw a possible graph of* $y = f(x)$ *given the following information about its derivative.*

(1) $f'(x) > 0$ *for* $x < 2$
(2) $f'(x) < 0$ *for* $x > 2$
(3) $f'(x) = 0$ *for* $x = 2$

Example 2.3 (2.3 Text 51)**.** *The figure shows the graphs of* f *and* f'*. Determine which is which.*



3. Derivative Formulas

(1) **Constant Function:** $\dfrac{d}{dx}(k) =$

(2) **Linear Function:** $\dfrac{d}{dx}(mx+b) =$

(3) **Power Function:** $\dfrac{d}{dx}(x^n) =$

Example 3.1. *Prove formula* **??** *using the definition of the derivative for the case where n is a positive integer and using the Binomial Theorem :*

$$(a+b)^n = a^n + na^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n.$$

Example 3.2.

Use the formulas to find the derivatives of the functions.

(1) $f(x) = \pi^2$

(2) $g(x) = \dfrac{x+4}{5}$

(3) $h(x) = x^2$

(4) $k(x) = \sqrt[3]{x}$

(5) $k(x) = \dfrac{1}{3x^4}$