1. Chapter 2 Section 4: Interpretations of the Derivative

Recall that another name for the derivative of f with respect to x is the instantaneous rate of change or f with respect to x and the definitions: $f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$ and $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Remarks 1.1.

(1) *Notice these limits could also be written* $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$, *where* Δ *is representing "change". If this limit exists and if* Δx *is very close to zero, then* $f'(x) \approx \frac{\Delta y}{\Delta x}$ *(no limit on right).*

(2) *The "Liebniz" notation for the derivative of* y *with respect to* x, $\frac{dy}{dx}$, *provides a better hint of the relationship between the derivative and* $\frac{\Delta y}{\Delta x}$. *The parts of this notation* dy *and* dx *are called* **infinitesimals** *and are often thought of as objects that are infinitely close to zero. This is not a precise definition, just a more intuitive explanation.*

(3) *The "Liebniz" notation for the derivative of* y *with respect to* x, $\frac{dy}{dx}$, *also provides a better hint of the units of the derivative. The units of* $\frac{dy}{dx}$ *are the units of* y *over the units of* x.

Example 1.1 (2.6 WP Homework Questions 1, Text 6)**.** *High internet download speeds generally cost more in fees. Let* $c = f(s)$ *be the monthly cost in dollars for a speed of* s *megabytes per second, Mbps.*

(1) *In words with units, give interpretations of*

(a) $f(10) = 40$

(b) $f'(10) = 2$

(2) *Is* $f(s)$ *an increasing or decreasing function of* s*?*

Example 1.2 (2.6 Text 50)**.** *The Arctic Sea ice extent, the area of sea covered by ice, grows over the winter months, typically from October to March. Let $F(t)$ be the Arctic Sea ice extent, in millions of square kilometers, as a function of time, t, in days since January 1, 2018. The $F'(t) = 0.08$ on January 1, 2019.*

(1) Give the units of the 0.08, and interpret the number in practical terms.

(2) Estimate ΔF, the change in F, between January 1 and January 4, 2019. Explain what this tells us about the Arctic Sea ice.