1. Chapter 3 Section 10: The Mean Value Theorem

Theorem 1.1 (The Mean Value Theorem)**.** *Let f be a function satisfying the following properties:*

(1) f is continuous on the interval $[a, b]$ and
(2) f is differentiable on the interval (a, b).

(The above two properties are the **hypotheses** *of the Mean Value Theorem Theorem and the following is the* **conclusion***.)*

Then there is a number c in (a, b) such that __________________.

Equivalently, there is a number c in (a, b) such that __________________.

Corollary 1.1.1 (Rolle's Theorem)**.** *Let f be a function satisfying the following properties:*

(1) f is continuous on the interval $[a, b]$,
(2) f is differentiable on the interval (a, b), and
(3) $f(a) = f(b)$.

(The above three properties are the **hypotheses** *of Rolle's Theorem and the following is the* **conclusion***.)*

Then there is a number c in (a, b) such that __________________.

2. Examples

Example 2.1. *Determine if f satisfies the conditions of the Mean Value Theorem. If so, find all c that satisfy the conclusion of the Mean Value Theorem. If not, which property/properties is/are not satisfied?*

$f(x) = \dfrac{x}{x-2}$, $[0, 1]$

Example 2.2. *Determine if f satisfies the conditions of the Mean Value Theorem. If so, find all c that satisfy the conclusion of the Mean Value Theorem. If not, which property/properties is/are not satisfied?*

$f(x) = \dfrac{x}{x-2}$, $[1, 3]$

3. The Increasing Function Theorem

Theorem 3.1 (The Increasing Function Theorem)**.** *Suppose that f is continuous on $a \le x \le b$ and differentiable on $a < x < b$.*

- *If $f'(x) > 0$ on $a < x < b$, then f is __________________ on $a \le x \le b$.*
- *If $f'(x) \ge 0$ on $a < x < b$, then f is ________________________ on $a \le x \le b$.*

4. The Constant Function Theorem

Theorem 4.1 (The Constant Function Theorem)**.** *Suppose that f is continuous on $a \le x \le b$ and differentiable on $a < x < b$. If $f'(x) = 0$ on $a < x < b$, then f is constant on $a \le x \le b$.*

5. The Racetrack Principle

Theorem 5.1 (The Racetrack Principle)**.** *Suppose that g and h are continuous on $a \le x \le b$ and differentiable on $a < x < b$, and that $g'(x) \le h'(x)$ for $a < x < b$.*

- *If $g(a) = h(a)$, then _______________ for $a \le x \le b$.*
- *If $g(b) = h(b)$, then _______________ for $a \le x \le b$.*

Example 5.1 (Section 3.10, Text Problem 20)**.** *Use the Racetrack Principle and the fact that $\sin(0) = 0$ to show that $\sin x \le x$ for all $x \ge 0$.*