1. Chapter 4 Section 1: Local Extrema

Definitions 1.1. *In the definitions below, assume $y = f(x)$ is a function with domain D, and c is a number in the domain of f.*

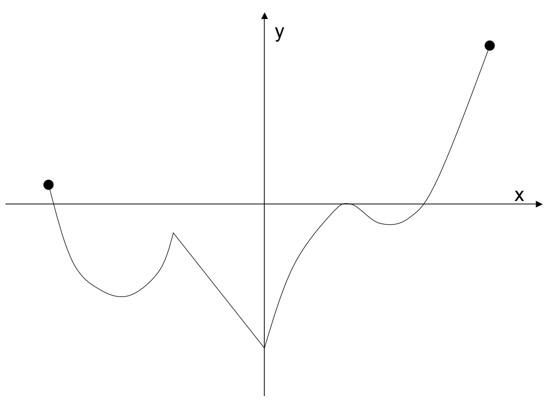
(1) *f has a* **local maximum** *or* **relative maximum** *at $x = c$ if $f(c) \geq f(x)$ for x "close enough" to c. $f(c)$ is the* **local maximum (value)**.

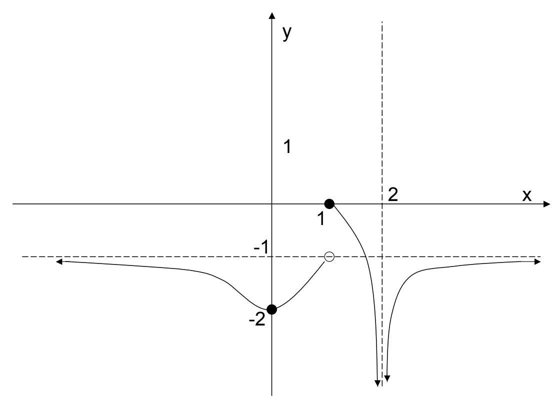
(2) *f has a* **local minimum** *or* **relative minimum** *at $x = c$ if $f(c) \leq f(x)$ for x "close enough" to c. $f(c)$ is the* **local minimum (value)**.

(3) *"Close enough" to c means there is an open interval around c where the statement is true for all values in that open interval and in the domain of the function. This open interval can be very small.*

(4) *The local minimum and local maximum values are called the* **local extreme values** *of f.*

Example 1.1. *Find all local extrema and where they occur on each graph.*





Theorem 1.1 (Fermat's Theorem). *If f has a local extreme at c, then $f'(c) = 0$ or $f'(c)$ is undefined.*

Definition 1.1. *A* **critical point** *of a function f is a number p in*

the ____________ *of f such that either* ____________ *or* ____________.

A **critical value** *of f is $f(p)$ where p is a critical point.*

Example 1.2 (2.1 WP Homework Question 4, Text 17)**.** *Let* $g(x) = xe^{-5x}$*. Find all critical points of the function.*

Recall

Theorem 1.2. *Suppose* f *is a function that is differentiable on the interval* (a, b)*.*

1. $f'(x) = 0$ *for all* x *in the interval* (a, b) *if and only if* f *is a*

   __________________________ *function on* (a, b)*.*

2. $f'(x) > 0$ *for all* x *in the interval* (a, b)*, except possibly a finite number of points,*

   if and only if f *is a* __________________________ *function on* (a, b)*.*

3. $f'(x) < 0$ *for all* x *in the interval* (a, b)*, except possibly a finite number of points,*

   if and only if f *is a* __________________________ *function on* (a, b)*.*

Example 1.3 (2.1 WP Homework Question 4, Text 17, Continued)**.** *Let* $g(x) = xe^{-5x}$*. Find all intervals where the function is increasing and decreasing.*

2. First Derivative Test for Local Extrema

First Derivative Test:

Find all critical numbers of f. Keep in mind that all critical numbers must be in the domain of f.

1. If f' is positive to the left of c and negative to the right of c, then f has a...

2. If f' is negative to the left of c and positive to the right of c, then f has a...

3. If f' does not change signs at c, then f...

3. Finding **LOCAL** Extrema Using the First Derivative Test

1. Find the domain of f.
2. Find all critical numbers of f. These will be your potential extrema.
3. Place all critical numbers AND values where f is undefined on a number line. These numbers will separate the number line into intervals.
4. Determine the sign of f' on each interval on the number line.
5. Use the information in 4 to determine intervals where f is increasing, where it is decreasing, and where local extremes occur.

Example 3.1 (2.1 WP Homework Question 4, Text 17, Continued)**.** *Let* $g(x) = xe^{-5x}$. *Find the local extrema of* f.

4. The Second Derivative Test for Local Extrema

Recall

Theorem 4.1. *Suppose f is a function that is twice differentiable on the interval (a, b).*

1. $f''(x) = 0$ *for all x in the interval (a, b) if and only if f is a linear function on (a, b).*
2. $f''(x) > 0$ *for all x in the interval (a, b), except possibly a finite number of points, if and only if f is concave up on (a, b).*
3. $f''(x) < 0$ *for all x in the interval (a, b), except possibly a finite number of points, if and only if f is concave down on (a, b).*

Definition 4.1. *$x = p$ is called an* **inflection point** *of f is p is in the domain of f, f is continuous at p, and f changes concavity at p.*

Example 4.1. *Let $g(x) = xe^{-5x}$. Find the inflection point(s) of f.*

Theorem 4.2 (Second Derivative Test)**.** *Suppose $y = f(x)$ is such that $f'(c) = 0$ (and f is twice differentiable around c).*

(1) *If $f''(c) > 0$ then* ______________________________

(2) *If $f''(c) < 0$ then* ______________________________

(3) *If $f''(c) = 0$ then* ______________________________

Example 4.2. *Can the Second Derivative Test be used to find all the local extrema of the following function? Find any local extrema the Second Derivative Test can be used for and find the rest using the First Derivative Test.*

$h(x) = 4x^5 - 5x^4 - 40x^3$

Example 4.3. *Suppose the graph below is the graph of $f'(x)$. Where are the critical points of f, the local minimums of f, and the local maximums of f on the interval $[-5, 10]$. If the graph is instead $f''(x)$, find the inflection points of f on the interval $[-5, 10]$.*