1. Chapter 4 Section 6: Rates and Related Rates

2. Related Rates

Example 2.1. *(https://www.geogebra.org/3d/vmwypzyh)* $\frac{dV}{dt}$ *in terms of* r, h, $\frac{dr}{dt}$ *and* $\frac{dh}{dt}$. *What is* $\frac{dh}{dt}$ *when* $\frac{dV}{dt} = 10$, $r = 3$, $h = 5$, $\frac{dr}{dt} = 8$*?*

$$V = \frac{1}{3}\pi r^2 h$$

Example 2.2. *A cone is growing at a rate of 10* in^3/min *and the radius is growing at a rate of 8* in/min. *Find how fast the height is changing if the radius is 3 in and height is 5 in. Is the height increasing or decreasing?*

Example 2.3 (4.6 WP Homework Question 9, Text 58 parts a and b)**.** *Coroners estimate time of death using the rule of thumb that a body cools about* $2°F$ *during the first hour after death and about* $1°F$ *for each additional hour. Assuming an air temperature of* $68°F$ *and a living body temperature of* $98.6°F$*, the temperature* $T(t)$ *in* $°F$ *of a body at a time* t *hours since death is given by*

$$T(t) = 68 + 30.6e^{-kt}.$$

(1) For what value of k *will the body cool by* $2°F$ *in the first hour?*

(2) Using the value of k *found in part 1, after how many hours will the temperature of the body be decreasing at a rate of* $1°F$ *per hour?*

3. Related Rates: Combining or Deriving Equations

General Steps:

Step 1. Read the problem quickly for a basic overview. Draw pictures and write down the formulas that may be relevant.
Step 2. Reread the problem carefully and express all information from the problem mathematically. Use variables to represent any quantity that changes. Numbers may be used for quantities that remain constant. Be sure to note which quantities change and which do not.
Step 3. Find equations that relate the quantities discussed in the problem.
Step 4. Use implicit differentiation to get an equation that relates rates of changes from the equation in Step 3.
Step 5. Plug in the values that were variable.
Step 6. Re-read the problem and answer the question.

Example 3.1 (4.6 Text 30)**.** *Car A is driving south, away from an intersection. Car B is approaching the intersection and is movnig west. At what rate is the distance between the cars changing at the instant when car A is* 40 *miles from the intersection and traveling at* 55 *mph and car B is* 30 *miles from the intersection and traveling at* 45 *mph? Are the cars getting closer together or farther apart at this time?*

Example 3.2 (4.6 Text 35a)**.** *A* 10 ft *ladder leans against a vertical wall and the bottom of the ladder slides away from the wall at a rate of* 0.5 ft/sec*. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is* 4 ft *from the wall?*