1. Chapter 4 Section 7: l'Hopital's Rule, Growth, and Dominance

First we review some of the indeterminate forms that we have addressed earlier in the semester.

(1) $\dfrac{0}{0}$

(2) $\pm\dfrac{\infty}{\infty}$

(3) $\pm\infty \cdot 0$

(4) $\infty - \infty$

Example 1.1. $\displaystyle\lim_{x\to\infty} \frac{3x^4 + 2x^2}{x^4 + 16}$

Theorem 1.1 (L'Hopital's Rule)**.** *Consider* $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)}$*. If this limit has the indeterminate form* $\frac{0}{0}$ *or* $\pm\frac{\infty}{\infty}$*, then*

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

To use l'Hospital's Rule for any other indeterminate form, the function must be rewritten so that it is in one of the indeterminate forms $\frac{0}{0}$ *or* $\pm\frac{\infty}{\infty}$.

Example 1.2. *Redo using l'Hospital's Rule.* $\displaystyle\lim_{x\to\infty} \frac{3x^4 + 2x^2}{x^4 + 16}$

Example 1.3. $\lim_{x \to 0^+} \tan x \ln x$

Example 1.4. $\lim_{x \to \infty} (x \cos(1/x) - x)$

2. Indeterminate Forms Continued

There are three more indeterminate forms that we will need to recognize that are new for this section.

(5) 0^0 (6) ∞^0 (7) 1^∞

Example 2.1. $\lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt}$

Example 2.2 (4.8 text 84)**.** $\lim_{t\to 0}\left(\frac{3^t+5^t}{2}\right)^{1/t}$