1. Chapter 5 Section 1: How Do We Measure Distance Traveled - Positive velocity?

Example 1.1. *The speedometer readings for a motorcycle at 12-second intervals are given in the table. t is in seconds and v is in ft/s.*

t	0	12	24	36	48	60
$v(t)$	30	28	25	22	24	27

(1) *Estimate the distance traveled by the motorcycle during this time period using velocities at the beginning of each subinterval.*

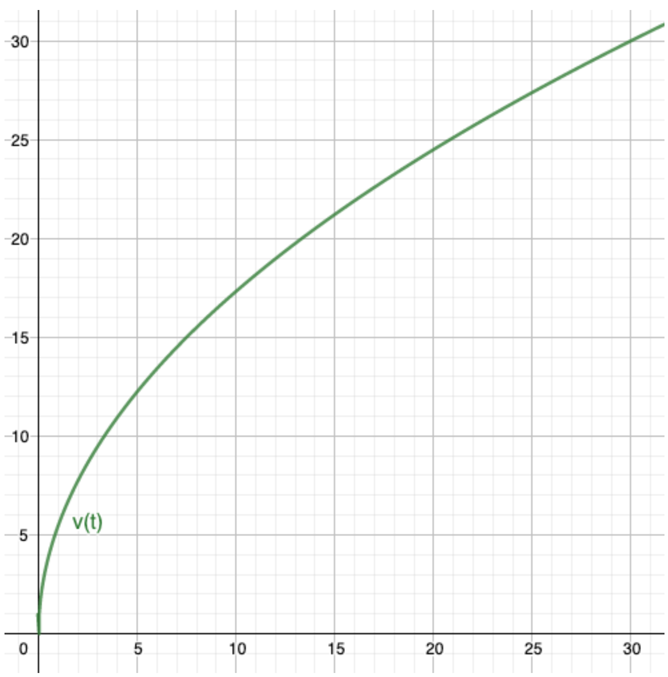
(2) *Estimate the distance traveled by the motorcycle during this time period using velocities at the end of each subinterval.*

Example 1.2. *Desmos https://www.desmos.com/calculator/hlfrzv3x7f*

and https://www.desmos.com/calculator/tzjvanvsmi

The graph of the velocity, v, in feet per second of an object at time t second after noon yesterday is given below and in the linked Desmos graphs. In the Desmos links, the value of S is the estimate of the distance traveled using d second intervals to and using the left or the right velocities in each interval. The area of each rectangle in the graphs represent the velocity used for that interval times the time covered in that interval (d). The rectangles are drawn beyond the interval $[0, 30]$, when d is chosen to be larger than 15. Ignore the rectangles beyond $t = 30$!

Explore the estimated values of the distance traveled using different values of d and using values at the left or right side of the intervals. Which estimates would you expect to be better? Which worse? Which would be overestimates and which underestimates?



Remark 1.1. *The total distance traveled in the previous example, would be exactly equal to the area under the velocity curve.*

2. How Do We Measure Distance Traveled - positive and negative velocity?

Example 2.1. *The velocity in feet per second of an object moving along a straight line is given by the values in the table where t is in second.*

t	0	2	4	6
$v(t)$	5	3	-1	-2

(1) *Estimate the change in position over the interval $0 \leq t \leq 6$ seconds using the velocities at the end of each interval.*

(2) *Estimate the total distance traveled over the interval $0 \leq t \leq 6$.*

Remarks 2.1. *Suppose we are given the graph of velocity, v, of an object at time t.*

(1) *The change in position of an object (displacement) over the interval $[a, b]$ can be found by adding the areas above the x-axis on the interval and subtracting the areas below the x-axis on the interval.*
(2) *The total distance traveled of an object over the interval $[a, b]$ can be found by adding both the areas above the x-axis on the interval and the areas below the x-axis on the interval. This is the same as using the absolute value of the velocity to find the distance traveled.*

3. Definitions and Notation

We assume for this section that f is a continuous function on the interval $[a, b]$ and n is a positive integer.

(1) We may subdivide the interval $[a, b]$ into n subintervals. Denote the endpoints of the subintervals $x_0, x_1, x_2, \ldots, x_n$ where $a = x_0$, $b = x_n$ and $x_{i-1} < x_i$. This is called a **partition** of the interval $[a, b]$.
(2) It is most common to choose the subintervals to all have the same width.
(3) $\Delta x_i = x_i - x_{i-1} =$ the width of the subinterval $[x_{i-1}, x_i]$.
(4) It is most common to choose the subintervals to all have the same width, so $\Delta x_i = \dfrac{b-a}{n}$
(5) x_i^* denotes a chosen number in the interval $[x_{i-1}, x_i]$.
(6) $\displaystyle\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ is a **Riemann Sum.**
(7) If $x_i^* = x_{i-1}$, then the Riemann sum is called the **left** Riemann sum.
(8) If $x_i^* = x_i$, then the Riemann sum is called the **right** Riemann sum.
(9) If $x_i^* = \dfrac{x_i - x_{i-1}}{2}$, then the Riemann sum is called the **midpoint** Riemann sum.
(10) If x_i^* is where the maximum occurs on $[x_{i-1}, x_i]$, then the Riemann sum is called the **upper** Riemann sum.
(11) If x_i^* is where the minimum occurs on $[x_{i-1}, x_i]$, then the Riemann sum is called the **lower** Riemann sum.