1. Chapter 5 Section 2: The Definite Integral

Recall from 5.1 that the Riemann sum, $\sum_{i=1}^{n} f(x_i^*)\Delta x_i$ approximates the area between the graph of a positive valued function and the x-axis over the interval $[a, b]$ or the total distance traveled over the interval $[a, b]$ if f is a positive valued velocity function.

We can use the limit of Riemann sums as the widths of the subintervals approached zero to find the exact value of the area. This limit is also can be used to find distance traveled from velocity. This is just two example of where the limit of a Riemann sum can be applied.

Definition 1.1. *The* **definite integral of** f **from** a **to** b *is defined by*

$$\int_a^b f(x)\,dx = \lim_{max(\Delta x_i)\to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

Note that when the subintervals are chosen so the width of each is $\frac{b-a}{n}$*, then this is equivalent to the following limit using right Riemann sums.*

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

(1) $\int$ is called the **integral sign**.
(2) a and b are the **limits of integration**, with b being the **upper limit** while a is the **lower limit**.
(3) $f(x)$ is the **integrand**.
(4) The process of finding the integral is **integration**.

Remark 1.1. *If* $f(x) > 0$ *on* $[a, b]$*, then* $\int_a^b f(x)\,dx$ *is the exact area between the curve and the* x*-axis over the interval* $[a, b]$*.*

Example 1.1. *Let* $f(x) = \sqrt{25 - x^2}$.

(1) *Sketch the graph of* $f(x) = \sqrt{25 - x^2}$. *Then sketch the left-hand Riemann sum with* $n = 5$ *that would be used to estimate* $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$.

(2) *Use the left-hand Riemann sum with* $n = 5$ *to estimate* $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$.

(3) *Use an online calculator to find* $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$.

2. 5.2 Definite Integral Versus Riemann Sum

Example 2.1 (5.2 text 64). *Express the limit as a definite integral for the given* t_i.

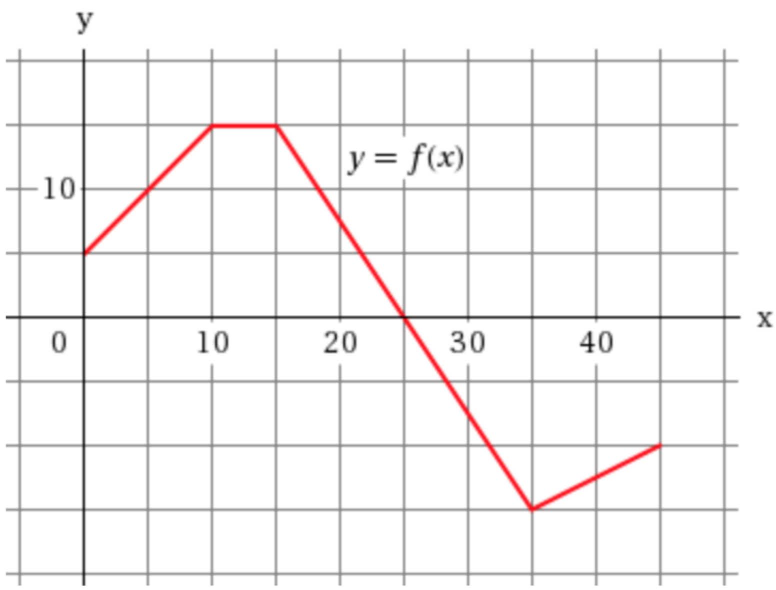
$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{n} e^{1+i/n}; \qquad t_i = 1 + \frac{i}{n}$$

3. Properties

(1) If $f(x) > 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is the exact area between the curve and the x-axis over the interval $[a, b]$.

(2) If $f(x) < 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is -1 times the exact area between the curve and the x-axis over the interval $[a, b]$.

Example 3.1. *Use the graph to find the following integrals.*



(1) $\int_0^{10} f(x)\,dx$

(2) $\int_{25}^{35} f(x)\,dx$

Properties continued

(3) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(4) $\int_a^a f(x)\,dx = 0$

(5) $\int_a^b c\,dx = c(b-a)$

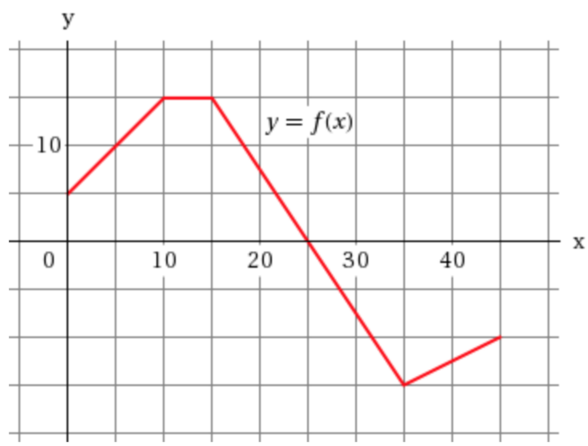
(6) $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

(7) $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(8) $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

(9) $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

Example 3.2. *(continued) Use the graph to find the following integral.*



$$\int_{15}^{35} f(x)\,dx$$

Example 3.3. *Evaluate the integral by interpreting it in terms of areas. Compare to example 1.*

$$\int_{-5}^{5} \sqrt{25 - x^2}\,dx$$