1. Chapter 5 Section 3: The Fundamental Theorem and Interpretations

In section 5.1 we learned If $v(t)$ is the velocity function and its units are feet per second, then

$$\int_a^b v(t)\,dt = [\text{ Change in position from } s(a) \text{ to } s(b)] = s(b) - s(a)$$

and the units will be in feet.

Recall from section 2.4: The "Liebniz" notation for the derivative of y with respect to x, $\dfrac{dy}{dx}$, provides a better hint of the relationship between the derivative and $\dfrac{\Delta y}{\Delta x}$. The parts of this notation dy and dx are called **infinitesimals** and are often thought of as objects that are infinitely close to zero. This is not a precise definition, just a more intuitive explanation.

This concept continues to be useful in determining units in integration. We still think of dx as an infinitesimally small change in x, so its units would be the units of x. In the above example, then, the units of $\displaystyle\int_a^b v(t)\,dt$ is the same as the units of $v(t)$ times the units of dt. In particular, the units in the above example are $\frac{feet}{second} \times seconds = feet$.

Example 1.1 (5.3 Text problem 3)**.** *If $f(x)$ is measured in pounds and x is measured in feet, what are the units of $\displaystyle\int_a^b f(x)\,dx$?*

2. The Definite Integral of a Rate of Change

When $f(t) = F'(t)$, so f represents the rate of change of F with respect to x, then $\int_a^b f(t)\,dt$ represent the total change in F between $t = a$ and $t = b$.

Theorem 2.1 (The Fundamental Theorem of Calculus)**.** *If f is a continuous function on the interval $[a, b]$ and $f(t) = F'(t)$, the*

$$\int_a^b f(t)\,dt = F(b) - F(a).$$

Example 2.1 (5.3 Text problem 34)**.** *The table gives annual US emissions, $H(t)$, of "super greenhouse gases," in millions of metric tons of carbon-dioxide equivalent. Let t be in years since 2000. What are the units and the meaning of $\int_2^{14} H(t)\,dt$?*

Year	2002	2004	2006	2008	2010	2012	2014
$H(t)$	142.3	139.6	144.1	157.5	164.0	170.1	180.1

Example 2.2 (5.3 Text problem 34, continued)**.** *The table gives annual US emissions, $H(t)$, of "super greenhouse gases," in millions of metric tons of carbon-dioxide equivalent. Let t be in years since 2000. Given the table below, estimate* $\int_2^{14} H(t)\,dt$.

Year	2002	2004	2006	2008	2010	2012	2014
$H(t)$	142.3	139.6	144.1	157.5	164.0	170.1	180.1

3. Antiderivatives

Definitions 3.1.
(1) A function F is called an **antiderivative** *of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.*
(2) Theorem: If F is an antiderivative of f on an interval I and C is any constant, then $F(x) + C$ also defines an antiderivative of f on I.
(3) If F is an antiderivative of f, then the **most general antiderivative** *is $F(x) + C$, where C is understood to represent any constant.*
(4) The most general antiderivative is also called the **indefinite integral** *of f and we will use this language in later chapters.*
(5) The constant C is called the **constant of the antiderivative** *or* **integration**.

Example 3.1. *Let* $f(x) = x^2$*. Determine an antiderivative for* f*.*

Example 3.2. *Use Example 3.1 and the Fundamental Theorem of Calculus to find* $\displaystyle\int_{-1}^{2} x^2\,dx$*.*