1. Chapter 5 Section 4: Theorems About Definite Integrals

Recall

Definition 1.1. *The* **definite integral of** f **from** a **to** b *is defined by*

$$\int_a^b f(x)\,dx = \lim_{max(\Delta x_i)\to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

Note that when the subintervals are chosen so the width of each is $\dfrac{b-a}{n}$*, then this is equivalent to the following limit using right Riemann sums.*

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i$$

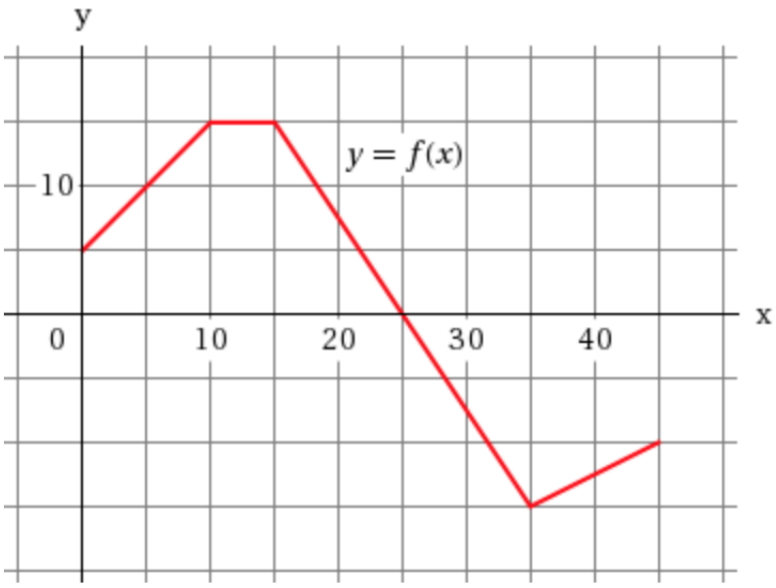
2. Properties

(1) If $f(x) > 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is the exact area between the curve and the x-axis over the interval $[a, b]$.

(2) If $f(x) < 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is -1 times the exact area between the curve and the x-axis over the interval $[a, b]$.

(3) $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Example 2.1. *Use the graph to find the following integral.*



$$\int_{10}^{0} f(x)\,dx =$$

Properties continued

(4) $\displaystyle\int_a^a f(x)\,dx = 0$

(5) $\displaystyle\int_a^b c\,dx = c(b-a)$

(6) $\displaystyle\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

(7) $\displaystyle\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(8) $\displaystyle\int_a^b [f(x)-g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

(9) $\displaystyle\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

(10) If $f(x) \leq g(x)$ for all x in $[a, b]$, then $\displaystyle\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.

(11) If $m \leq f(x) \leq M$ for all x in $[a, b]$, then $m(b-a) \leq \displaystyle\int_a^b f(x)\,dx \leq M(b-a)$.

(12) If f is a continuous, even function; then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

(13) If f is a continuous, odd function; then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.

Example 2.2. . *Suppose* $\displaystyle\int_3^5 f(x)\,dx = 6$, $\displaystyle\int_0^5 f(x)\,dx = -1$, *and* $\displaystyle\int_3^{10} f(x)\,dx = 2$.

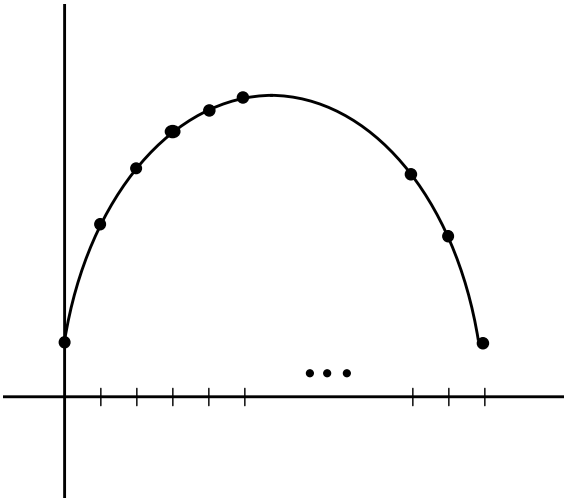
(1) Find $\displaystyle\int_5^{10} f(x)\,dx$.

(2) If f is an even function, find $\displaystyle\int_{-3}^{3} f(x)\,dx$

3. Average Function Value

The average value of f over the interval $[a, b]$ is defined as

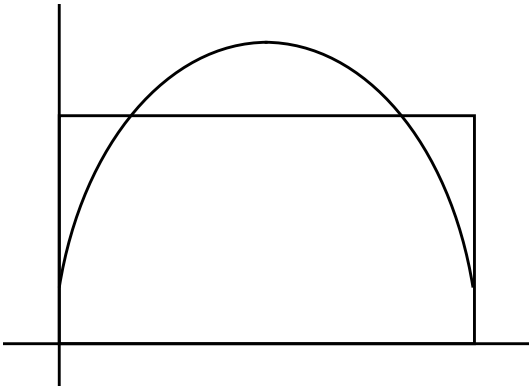
$$f_{ave} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} [f(x_i^*)]}{n} =$$



4. Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, then there is a number c in $[a, b]$ with

$$\int_a^b f(x)\,dx = f(c)(b-a)$$



Example 4.1. . *Suppose* $\int_3^5 f(x)\,dx = 6$, $\int_0^5 f(x)\,dx = -1$, *and* $\int_3^{10} f(x)\,dx = 2$.

Find the average value of $f(x)$ *on the interval from* $x = 5$ *to* $x = 10$.

Example 4.2 (5.4 WP Homework Questions 2, Text 8)**.** *Find the average value of the function* $g(t) = e^t$ *over the interval* $[0, 10]$. *Illustrate the average value on a graph.*