1. Chapter 6 Section 1: Antiderivatives Graphically and Numerically

In Chapter 5 we discussed the Definite integral and have touched on the relationships between the antiderivative and the definite integral. We re-state that information and dig further into the anti derivative in this section.

Definition 1.1. *A function F is called an* **antiderivative** *of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.*

Theorem 1.1 (Fundamental Theorem of Calculus)**.** *If f is continuous on $[a, b]$ and $f(t) = F'(t)$, then*

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

.

Example 1.1. *Select all of the following functions that are antiderivatives for $f(x) = x^4$.*

(1) $F(x) = \frac{1}{5}x^5 - \sqrt{3}$

(2) $F(x) = x^5$

(3) $F(x) = \frac{1}{5}x^5 + 4$

(4) $F(x) = \frac{1}{5}x^5$

(5) $F(x) = \frac{1}{4}x^5$

Example 1.2. *Select all of the following functions, F, that are antiderivatives for $f(x) = x^4$ AND are such that $F(0) = 4$*

(1) $F(x) = \frac{1}{5}x^5 - \sqrt{3}$

(2) $F(x) = x^5$

(3) $F(x) = \frac{1}{5}x^5 + 4$

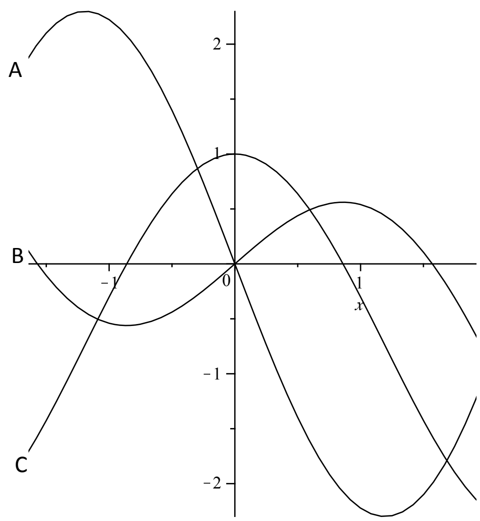
(4) $F(x) = \frac{1}{5}x^5$

(5) $F(x) = \frac{1}{4}x^5$

Notice in the prior examples that it is possible to find infinitely many antiderivatives of a single function, but additional condition(s) may restrict the answer to only one antiderivative.

2. Visualizing Antiderivatives

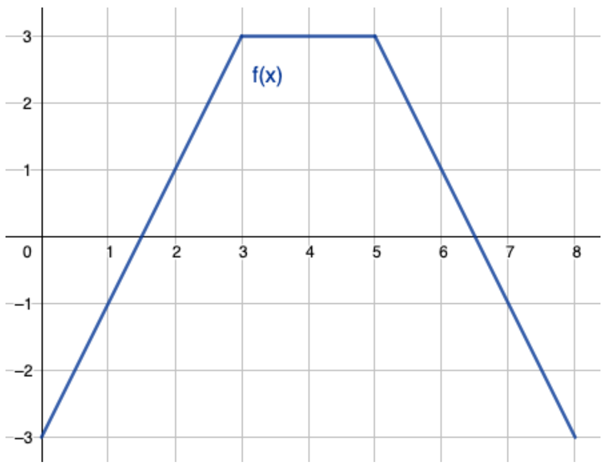
Example 2.1. *Which of the following curves could be the graph of an antiderivative of the graph of the function given by curve C?*



Example 2.2 (6.1 WP Homework Questions 5, Text 24)**.** *Estimate* $f(x)$ *for* $x = 2, 4, 6$, *using the given values of* $f'(x)$ *and the fact that* $f(0) = 21$. *Use an average of left- and right-hand sums to estimate the integrals.*

x	0	2	4	6
$f'(x)$	14	23	30	33

Example 2.3. *Suppose the graph below is the graph of $f(x)$. Sketch the graph of the antiderivative, $F(x)$, of f is it is given that $F(0) = 2$. Plot the points F has local extrema, changes in concavity, and the endpoints. If $F(0) = -2$, how does the graph change?*



Remark 2.1. *If F is an antiderivative of f on an interval I and C is any constant, then $F(x) + C$ also defines an antiderivative of f on I.*

Definitions 2.1.

1. *If F is an antiderivative of f, then the* **most general antiderivative** *or the* **family of antiderivatives** *is $F(x) + C$, where C is understood to represent any constant.*
2. *The most general antiderivative is also called the* **indefinite integral** *of f.*
3. *The most general antiderivative of $f(x)$ is also called the* **indefinite integral of** *f and is denoted $\int f(x)\,dx$.*

   In other words, if $F'(x) = f(x)$, then $\displaystyle\int f(x)\,dx = F(x) + C.$
4. *The constant C is called the* **constant of the antiderivative** *or* **integration**.