1. Chapter 6 Section 2: Constructing Antiderivatives Analytically

Recall where we left off in the previous section

Definition 1.1. *A function F is called an* **antiderivative** *of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.*

Theorem 1.1 (Fundamental Theorem of Calculus)**.** *If f is continuous on $[a, b]$ and $f(t) = F'(t)$, then*

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

.

Remark 1.1. *If F is an antiderivative of f on an interval I and C is any constant, then $F(x) + C$ also defines an antiderivative of f on I.*

Definitions 1.1.

1. *If F is an antiderivative of f, then the* **most general antiderivative** *or the* **family of antiderivatives** *is $F(x) + C$, where C is understood to represent any constant.*
2. *The most general antiderivative is also called the* **indefinite integral** *of f.*
3. *The most general antiderivative of $f(x)$ is also called the* **indefinite integral of** *f and is denoted $\int f(x)\,dx$.*

   In other words, if $F'(x) = f(x)$, then $\displaystyle\int f(x)\,dx = F(x) + C$.
4. *The constant C is called the* **constant of the antiderivative** *or* **integration**.

2. Rules for finding antiderivatives or indefinite integrals: Memorize

1. $n \neq -1$, $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$
2. $\int x^{-1}\,dx = \ln|x| + C$
3. $\int e^x\,dx = e^x + C$
4. $a > 0$ and $a \neq 1$, $\int a^x\,dx = \frac{1}{\ln a}a^x + C$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \sec^2 x\,dx = \tan x + C$
8. $\int \dfrac{1}{1+x^2}\,dx = \arctan x + C$
9. $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$

(10) $\int k\,dx = kx + C$

(11) $\int kg(x)\,dx = k\int g(x)\,dx$

(12) $\int g(x) \pm h(x)\,dx = \int g(x)\,dx \pm \int h(x)\,dx$

(13) $\int f(x)\,dx = F(x) + C$ then $\int f(kx)\,dx = \dfrac{1}{k}F(kx) + C$

3. Examples

Example 3.1. *Find the general antiderivative for* $f(x) = x^3 - 4e^{2x}$.

Example 3.2. *Suppose* $f(x) = x^3 - 4e^{2x}$. *Find the antiderivative of* f, $F(x)$, *such that* $F(0) = 4$

Example 3.3. *Find the indefinite integral* $\displaystyle\int \frac{x^4 + x - 4\sqrt[3]{x}}{x^2}\,dx$.

Example 3.4. *Find the area between* $y = f(x)$ *and the* x*-axis, over the interval* $1 \le x \le \sqrt{3}$.

$$f(x) = \frac{1}{1+x^2}$$

Example 3.5 (6.2 WP Homework Question 12, Text 99). *A car facing right moves along a straight line with velocity, in feet per second, given by*

$$v(t) = 8 - 2t \text{ for } t > 0.$$

1. *Describe the car's motion in words. (direction of motion?)*
2. *The car's position is measured from its starting point. When is it farthest forward? Backward?*
3. *Find s, the car's position measured from its starting point, as a function of time.*