MAD 3105, Section 1 - Quiz #3 and 4, Solutions

1. (6 points) Give precise definitions of the following:

(a) A Hamilton circuit in a graph \(G\): It is a closed path in \(G\) that contains all the vertices of \(G\) and does not repeat any vertices except the first and the last.

(b) A bipartite graph: A graph \(G\) such that \(V(G) = V_1 \cup V_2, V_1 \cap V_2 = \emptyset, V_1 \neq \emptyset\) and \(V_1 \neq \emptyset\) and such that each edge in \(G\) connects a vertex in \(V_1\) to a vertex in \(V_2\).

(c) An atom of a Boolean algebra: \(a\) is an atom if it cannot be written as \(a = x \lor y, x \neq a, y \neq a\).

2. (6 points) For the graph given below, give a Hamilton circuit or explain why none exists.

![Graph Image]

Solution: The graph is bipartite, with \(V_1 = \{a, b, c, d\}, V_2 = \{e, f, g\}\). Since \(|V_1| \neq |V_2|\), by theorem 4 of section 6.5, it follows that the graph cannot have a Hamilton circuit.

3. (8 points) Using only the axioms given above, show that in a Boolean algebra, if \(w \lor z = 1\) and \(w \land z = 0\), then \(z = w'\). Carefully state which of the axioms (or hypothesis) you have used in deducing your statements.

Solution:

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\begin{align*}
z &= z \lor 0 \quad &\text{(4a)} \\
   &= z \lor (w \land w') \quad &\text{(5b)} \\
   &= (z \lor w) \land (z \lor w') \quad &\text{(3a)} \\
   &= 1 \land (z \lor w') \quad &\text{(Hypothesis)} \\
   &= (w' \lor w) \land (w' \lor w) \quad &\text{(5a and 1a)} \\
   &= w' \lor w \land z \quad &\text{(5a and 1a)} \\
   &= w' \lor 0 \quad &\text{(Hypothesis)} \\
   &= w' \quad &\text{(4a)}. 
\end{align*}
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