0. State the following theorems:
   a) A theorem from Complex variables using the phrases "Cauchy-Riemann equations" and "if and only if". Be sure to write out the Cauchy-Riemann equations.
   b) A version of Cauchy’s integral theorem using the phrase "simply connected". Explain what "simply connected" means.

1. a) Give the first three non-zero terms of the Taylor expansion of \( \sin z \) at \( z = 0 \).
   b) Using multiplication of power series give the first three non-zero terms of the Taylor expansion of \( \sin^2 z \) at \( z = 0 \).
   c) Using division of power series give the first three non-zero terms of the Laurent expansion of \( \csc^2 z \) at \( z = 0 \).

2. Using residues find
   \[
   \int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 2x + 2)}.
   \]
   Give details on why the answer is correct.

3. a) Give the power series expansion at \( z = 0 \) of the function \( r(z) = \log(1 - z) + z + z^2/2 \) where \( \log \) is the principal branch of the logarithm.
   b) By estimating terms of the power series show that for \( |z| < 1 \),
      \[
      |r(z)| \leq \frac{|z|^3}{3(1 - |z|)}.
      \]
   c) Show that \( |r(z)| \leq \frac{2}{3}|z|^3 \) for \( |z| \leq \frac{1}{2} \).

4. How many zeros of \( z^4 - 5z + 1 \) are in \( |z| < 1 \)? How many in \( 1 < |z| < 2 \)?
   Prove your answer is correct.

5. For \( R \) real let \( \gamma_R \) be the contour consisting of the line segments \([R, R + Ri],[R + Ri, -R + Ri],[ -R + Ri, -R]\). By estimating each of the three integrals,
   show that
   \[
   \lim_{R \to \infty} \int_{\gamma_R} \frac{e^{iz}}{z} \, dz = 0
   \]