1. Let $\zeta = \exp(2\pi i/7)$ and $w = \zeta + \zeta^2 + \zeta^4$.
   a) Show that $1 + \zeta + \zeta^2 + \zeta^3 + \zeta^4 + \zeta^5 + \zeta^6 = 0$.
   b) Show that $w + \bar{w} = -1$.
   c) Show that $w^2 + w + 2 = 0$.

2. For each function below classify each isolated singularity in the finite plane and find the singular part and the residue at each singularity.
   \[
   \frac{z + 2}{(z^2 + 1)(2z - 1)^2} \quad \frac{\sin z - z + z^3/6}{z^4} \quad \frac{e^{1/z}}{z^2}
   \]

3. Compute
   \[
   \int_{-\infty}^{\infty} \frac{dx}{x^4 + 8i}.
   \]
   Give details.

4. Suppose $f(z)$ is an entire function such that $|f(z)| \leq 3|z|^6$ for $|z| > 5$. Show that $f$ is a polynomial of degree less than or equal to 6.

5. For $R > 0$ let $\gamma_R(\theta) = Re^{i\theta}$, $\pi/4 \leq \theta \leq 3\pi/4$. Show that
   \[
   \lim_{R \to \infty} \int_{\gamma_R} e^{iz} \, dz = 0.
   \]
   (Hint: Estimate $y$ from below on $\gamma_R$.)

6. Let $D = \{z \mid |z| < 1 \text{ and } \Im z > 0 \}$. Describe the image of $D$ under the following transformations: $ta^2 + \zeta^4$.
   a) $1/z$
   b) $z + 1$
   c) $\log z$ where $\log$ is the principal branch with the argument of $z$ in the interval $(-\pi, \pi)$.

7. State and prove Schwarz Lemma using the Maximum Principle.