2.18:
Claim: There is a nonempty perfect set E which contains no rational numbers.

Proof. Let $\{x_n\}_{n=0}^{\infty}$ be an enumeration of the rational numbers. Pick irrational numbers $a_{0,1} < b_{0,1} < x_0$ with $b_{0,1} - a_{0,1} < 1$ and let $E_0 = [a_{0,1}, b_{0,1}]$.

Suppose $E_0 \supset E_1 \supset \ldots \supset E_n$ have been constructed and that each

$$E_j = \bigcup_{k=1}^{2^j} [a_{j,k}, b_{j,k}]$$

where :

(i) For fixed j the intervals $[a_{j,k}, b_{j,k}]$, $k = 1, \ldots, 2^j$ are pairwise disjoint
(ii) The endpoints $a_{j,k}, b_{j,k}$ are irrational
(iii) Each $b_{j,k} - a_{j,k} < 2^{-j}$
(iv) $x_j \notin E_j$.

Set

$$E_{n+1} = \bigcup_{k=1}^{2^j} [a_{j,k}, b_{j,k}] \setminus I_k$$

where $I_k = (c_k, d_k)$ and
(1) c_k and d_k are irrational with $a_{j,k} < c_k < d_k < b_{j,k}$
(2) $c_k - a_{j,k} < \min(2^{-(n+1)}, |x_{n+1} - a_{j,k}|)$
(3) $b_{j,k} - d_k < \min(2^{-(n+1)}, |x_{n+1} - b_{j,k}|)$.
One can check that E_{n+1} is then a union of intervals which satisfy (i)-(iv).

Letting

$$E = \bigcap_{n=1}^{\infty} E_n$$

we have that E is closed and nonempty (intersection of nested sequence of nonempty compact sets). Furthermore E is perfect since the endpoints $a_{j,k}, b_{j,k}$ are all in E and from (iii) you can check that every point in E is a limit point of the set of endpoints (see for example Rudin's discussion of the middle-thirds Cantor set).

Finally, E has empty intersection with the rationals by (iv). □