10. A 1 kg mass is attached to a spring with spring constant $k=64$ N/m. With the mass on the spring at rest in the equilibrium position at time $t=0$, an external force $F(t)=(\frac{1}{2}t)$ N is applied until time $t_1=7\pi/16$ seconds, at which time it is removed. Assuming no damping, find the frequency and amplitude of the resulting oscillation.

11. A 1 kg mass is attached to a spring with spring constant $k=4$ N/m, and hangs in equilibrium. An external force $F(t)=(1+t+\sin 2t)$ N is applied to the mass beginning at time $t=0$. If the spring is stretched a length $(1/2+\pi/4)$ m or more from its equilibrium position, then it will break. Assuming no damping present, find the time at which the spring breaks.

12. A small object of mass 1 kg is attached to a spring with spring constant $k=1$ N/m. This spring-mass system is then immersed in a viscous medium with damping constant c. An external force $F(t)=(3-\cos t)$ N is applied to the system. Determine the minimum positive value of c so that the magnitude of the steady state solution does not exceed 5 m.

13. Determine a particular solution $\psi(t)$ of $my''+cy'+ky=F_0\cos\omega t$, of the form $\psi(t)=A\cos(\omega t-\phi)$. Show that the amplitude A is a maximum when $\omega^2=\omega_0^2-\frac{1}{2}(c/m)^2$. This value of ω is called the *resonant frequency* of the system. What happens when $\omega_0^2<\frac{1}{2}(c/m)^2$?

2.6.1 The Tacoma Bridge disaster

On July 1, 1940, the Tacoma Narrows Bridge at Puget Sound in the state of Washington was completed and opened to traffic. From the day of its opening the bridge began undergoing vertical oscillations, and it soon was nicknamed "Galloping Gertie." Strange as it may seem, traffic on the bridge increased tremendously as a result of its novel behavior. People came from hundreds of miles in their cars to enjoy the curious thrill of riding over a galloping, rolling bridge. For four months, the bridge did a thriving business. As each day passed, the authorities in charge became more and more confident of the safety of the bridge—so much so, in fact, that they were planning to cancel the insurance policy on the bridge.

Starting at about 7:00 on the morning of November 7, 1940, the bridge began undulating persistently for three hours. Segments of the span were heaving periodically up and down as much as three feet. At about 10:00 a.m., something seemed to snap and the bridge began oscillating wildly. At one moment, one edge of the roadway was twenty-eight feet higher than the other; the next moment it was twenty-eight feet lower than the other edge. At 10:30 a.m. the bridge began cracking, and finally, at 11:10 a.m. the entire bridge came crashing down. Fortunately, only one car was on the bridge at the time of its failure. It belonged to a newspaper reporter who had to abandon the car and its sole remaining occupant, a pet dog, when the bridge began its violent twisting motion. The reporter reached safety, torn and bleeding, by crawling on hands and knees, desperately clutching the curb of the bridge. His dog went down with the car and the span—the only life lost in the disaster.

There were many humorous and ironic incidents associated with the collapse of the Tacoma Bridge. When the bridge began heaving violently, the authorities notified Professor F. B. Farquharson of the University of Washington. Professor Farquharson had conducted numerous tests on a simulated model of the bridge and had assured everyone of its stability. The professor was the last man on the bridge. Even when the span was tilting more than twenty-eight feet up and down, he was making scientific observations with little or no anticipation of the imminent collapse of the bridge. When the motion increased in violence, he made his way to safety by scientifically following the yellow line in the middle of the roadway. The professor was one of the most surprised men when the span crashed into the water.

One of the insurance policies covering the bridge had been written by a local travel agent who had pocketed the premium and had neglected to report the policy, in the amount of $800,000, to his company. When he later received his prison sentence, he ironically pointed out that his embezzlement would never have been discovered if the bridge had only remained up for another week, at which time the bridge officials had planned to cancel all of the policies.

A large sign near the bridge approach advertised a local bank with the slogan "as safe as the Tacoma Bridge." Immediately following the collapse of the bridge, several representatives of the bank rushed out to remove the billboard.

After the collapse of the Tacoma Bridge, the governor of the state of Washington made an emotional speech, in which he declared "We are going to build the exact same bridge, exactly as before." Upon hearing this, the noted engineer Von Karman sent a telegram to the governor stating "If you build the exact same bridge exactly as before, it will fall into the exact same river exactly as before."

The collapse of the Tacoma Bridge was due to an aerodynamical phenomenon known as *stall flutter*. This can be explained very briefly in the following manner. If there is an obstacle in a stream of air, or liquid, then a "vortex street" is formed behind the obstacle, with the vortices flowing off at a definite periodicity, which depends on the shape and dimension of the structure as well as on the velocity of the stream (see Figure 1). As a result of the vortices separating alternately from either side of the obstacle, it is acted upon by a periodic force perpendicular to the direction of the stream, and of magnitude $F_0 \cos \omega t$. The coefficient F_0 depends on the shape of the structure. The poorer the streamlining of the structure; the larger the coefficient F_0, and hence the amplitude of the force. For example, flow around an airplane wing at small angles of attack is very smooth, so that the vortex street is not well defined and the coefficient F_0 is very small. The poorly streamlined structure of a suspension bridge is another

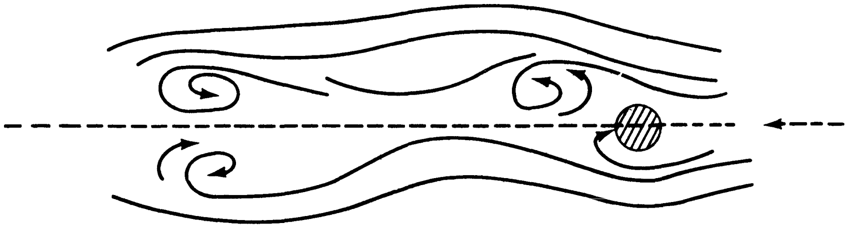

Figure 1

matter, and it is natural to expect that a force of large amplitude will be set up. Thus, a structure suspended in an air stream experiences the effect of this force and hence goes into a state of forced vibrations. The amount of danger from this type of motion depends on how close the natural frequency of the structure (remember that bridges are made of steel, a highly elastic material) is to the frequency of the driving force. If the two frequencies are the same, resonance occurs, and the oscillations will be destructive if the system does not have a sufficient amount of damping. It has now been established that oscillations of this type were responsible for the collapse of the Tacoma Bridge. In addition, resonances produced by the separation of vortices have been observed in steel factory chimneys, and in the periscopes of submarines.

The phenomenon of resonance was also responsible for the collapse of the Broughton suspension bridge near Manchester, England in 1831. This occurred when a column of soldiers marched in cadence over the bridge, thereby setting up a periodic force of rather large amplitude. The frequency of this force was equal to the natural frequency of the bridge. Thus, very large oscillations were induced, and the bridge collapsed. It is for this reason that soldiers are ordered to break cadence when crossing a bridge.

Epilog. The father of one of my students is an engineer who worked on the construction of the Bronx Whitestone Bridge in New York City. He informed me that the original plans for this bridge were very similar to those of the Tacoma Bridge. These plans were hastily redrawn following the collapse of the Tacoma Bridge.

2.6.2 Electrical networks

We now briefly study a simple series circuit, as shown in Figure 1 below. The symbol E represents a source of electromotive force. This may be a battery or a generator which produces a potential difference (or voltage), that causes a current I to flow through the circuit when the switch S is closed. The symbol R represents a resistance to the flow of current such as that produced by a lightbulb or toaster. When current flows through a coil of wire L, a magnetic field is produced which opposes any change in the current through the coil. The change in voltage produced by the coil is proportional to the rate of change of the current, and the constant of propor-