4.4. Curve-Sketching Techniques

Graphing Strategy

(1) Find from $y = f(x)$:

  (a) Domain: where is f defined? (Do NOT simplify before finding domain)

  (b) x-intercepts: set $y = 0$ and solve for x

  (c) y-intercepts: set $x = 0$ and solve for y

  (d) Asympotes

    (i) Vertical: find a so that $\lim_{x \to a} f(x) = \pm\infty$.

    (ii) Horizontal: find L so that $\lim_{x \to \pm\infty} f(x) = L$.

(2) Find from $y = f'(x)$:

  (a) Critical Numbers: where is $f'(x)$ equal to 0 or undefined in the domain of $f(x)$.

  (b) Horizontal and Vertical Tangents of $f(x)$

  (c) Intervals of increase and Interval of decrease of $f(x)$: use the sign of $f'(x)$

  (d) Local Extrema of $f(x)$

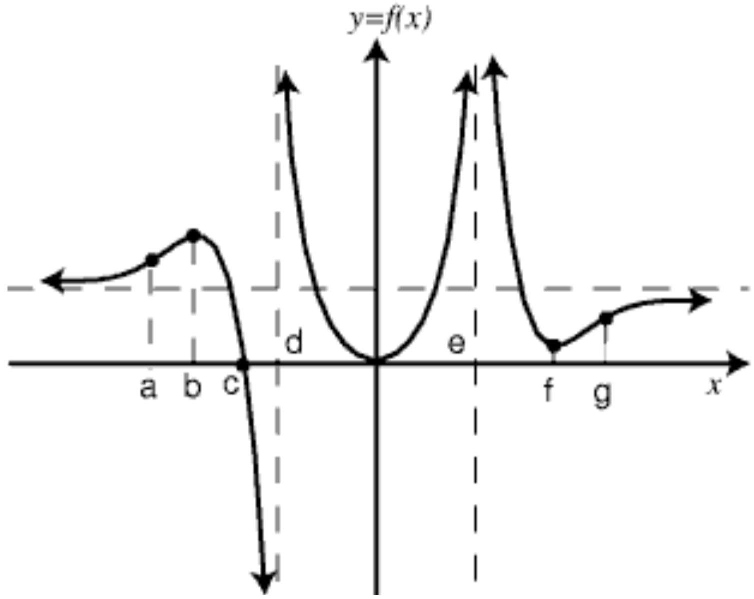
(3) Find from $y = f''(x)$:

  (a) Intervals of Concave Up and Concave down of $f(x)$: use the sign of $f''(x)$

  (b) Inflection Points of $f(x)$: where does $f''(x)$ change signs?

Examples

Example 4.4.1. *Assuming f', f'' exist, select ALL the correct choices for the graph.*



(1) $f''(x) > 0$ *on* $(-\infty, b) \cup (d, e) \cup (e, f)$

(2) $f''(x) < 0$ *on* $(b, d) \cup (f, \infty)$

(3) $f'(x) < 0$ *on* (c, d) *only*

(4) $f'(x) > 0$ *on* $(-\infty, c) \cup (d, e) \cup (e, \infty)$

(5) *the graph has inflection points at* $x = b$, $x = 0$, *and* $x = f$

(6) *the graph of* f *is concave downward on* $(a, d) \cup (g, \infty)$

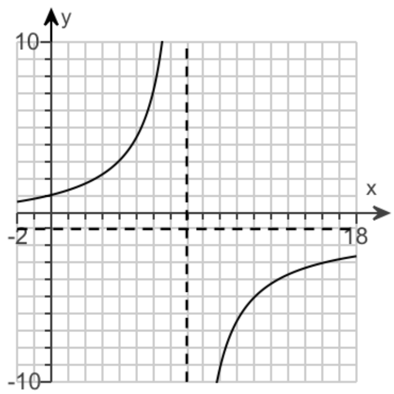
(7) $f(x)$ *has extremum at* $x = b$, $x = 0$, *and* $x = f$

(8) $f'(x)$ *has extremum at* $x = b$, $x = 0$, *and* $x = f$

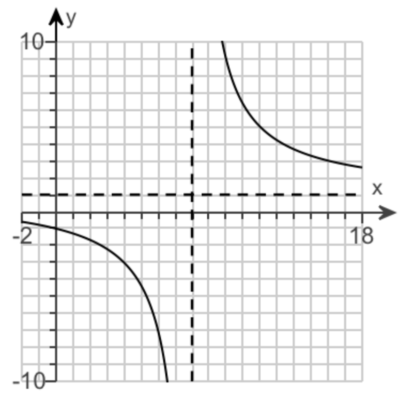
(9) $f'(x)$ *is increasing on* $(-\infty, c) \cup (d, e) \cup (e, \infty)$

(10) $f'(x)$ *is decreasing on* $(-\infty, d)$

Example 4.4.2. *Which graph below is the graph of* $f(x) = \dfrac{x+8}{x-8}$*. First find pertinent information including domain, asymptotes, intercepts, local extrema, and inflection points.*



(1)



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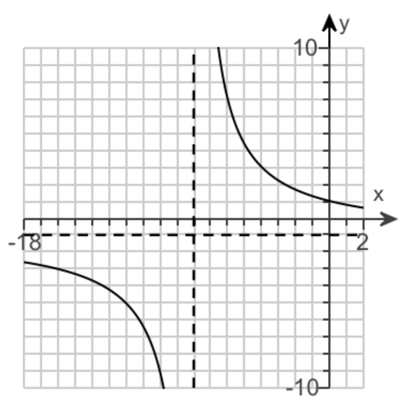
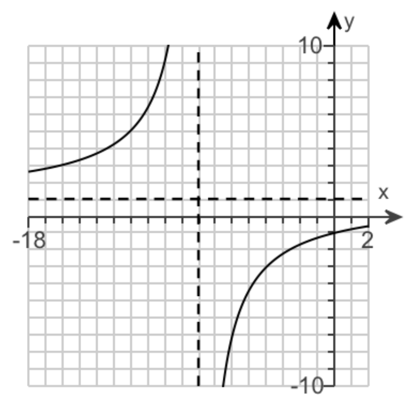
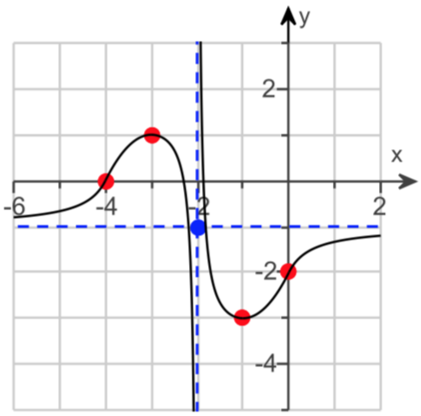


(2)

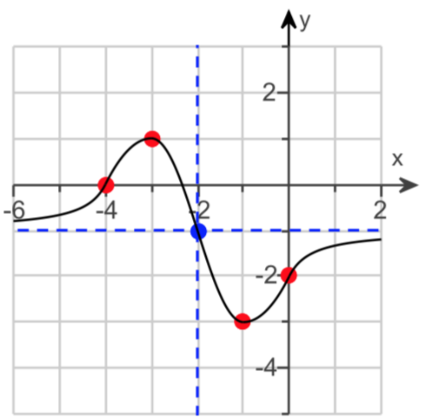


(4)

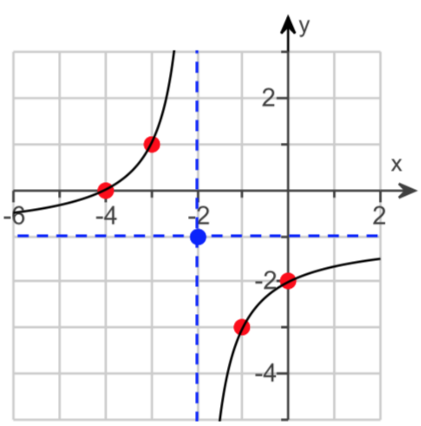
Example 4.4.3. *Use the given information to choose the correct graph of f.*
Domain: All real x, except $x = -2$
$f(-4) = 0$; $f(-3) = 1$; $f(-1) = -3$; $f(0) = -2$
$f'(x) > 0$ *on* $(-\infty, -2)$ *and* $(-2, \infty)$
$f''(x) > 0$ *on* $(-\infty, -2)$; $f''(x) < 0$ *on* $(-2, \infty)$
Vertical asymptote: $x = -2$; *Horizontal asymptote:* $y = -1$



(1)



(2)



(3)