Theorem 7.3.1. *Let $z = f(x, y)$ be a function of two variables.*

GIVEN:

(1) $f_x(a, b) = 0$ *and* $f_y(a, b) = 0$ *(*(a, b) *is a* ____________________ *for* f*)*

(2) *All second partial derivative exist around the point* (a, b)*.*

(3) $A = f_{xx}(a, b)$, $B = f_{xy}(a, b)$, $C = f_{yy}(a, b)$

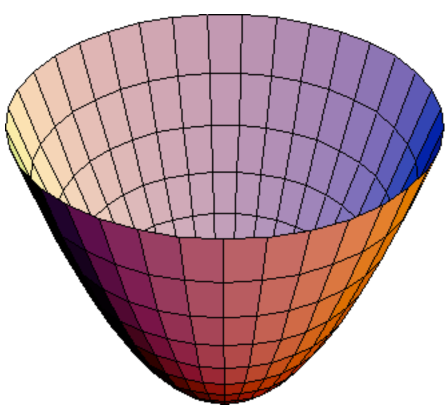
THEN:

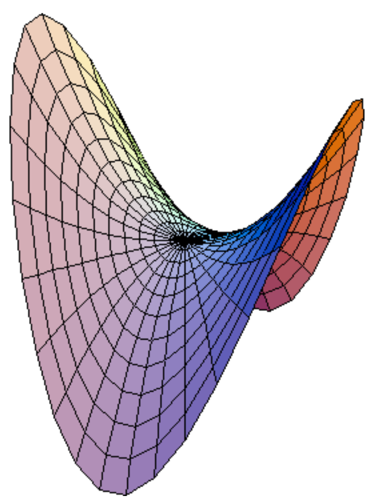
case 1: If $AC-B^2 > 0$ *and* $A < 0$, *then* $f(a, b)$ *is a* ____________________

case 2: If $AC-B^2 > 0$ *and* $A > 0$, *then* $f(a, b)$ *is a* ____________________

case 3: If $AC-B^2 < 0$, *then* f *has a* ____________________ *at* (a, b)*.*

case 4: If $AC - B^2 = 0$ *the test fails –* ____________________





Examples

Example 7.3.1. *Find all local extrema and saddle points of* $f(x, y) = 2x^2 - 2xy + y^2 - 4x + 6y - 3$

Example 7.3.2. *Find all local extrema and saddle points of* $f(x, y) = 8x + 6y - 17$

Example 7.3.3. *Find all local extrema and saddle points of* $f(x,y) = -2x^2 + 4xy - 3y^2 - 4x + 2y - 3$

Example 7.3.4. *Find all local extrema and saddle points of* $f(x,y) = xy + x - y$

Example 7.3.5. *Find all local extrema and saddle points of* $f(x, y) = 3y^2 - 2x^3 - 24x - 3y - 21$

Example 7.3.6. *Find all local extrema and saddle points of* $f(x, y) = 2x^3 - 2xy + 2y$

Example 7.3.7. *Let $z = f(x,y)$ have a critical point at $(3,-2)$ such that $f_{xx}(3,-2) = 32$, $f_{yy}(3,-2) = \frac{1}{2}$, and $f_{xy}(3,-2) = -4$. Then at $(3,-2)$, z has a*

a) local maximum
b) local minimum
c) saddle point
d) test fails

Example 7.3.8. *The cost function, C (in hundreds of dollars), of producing two products is $C(x,y) = 2x^2 + 3y^2 - 4xy + 4x - 8y + 20$, where x is the quantity of product A and y is the quantity of product B.*

(1) How many of each product should be produced to minimize cost

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(2) Find the minimum cost of producing these products.