Divide the interval from $x = a$ to $x = b$ into n equal subintervals of length $\Delta x = \dfrac{b-a}{n}$. Let c_k be the midpoint in the kth subinterval.

Then

$$\int_a^b f(x)\,dx \approx M_n = f(c_1)\Delta x + f(c_2)\Delta x + \ldots + f(c_n)\Delta x$$
$$M_n = \Delta x\left[f(c_1) + f(c_2) + \ldots + f(c_n)\right]$$

1. Use the rectangle (mid-point) rule, with n = 4, to approximate $\int_4^{12} \sqrt{x^2-1}\,dx$. Compute your answer to three decimal places.

$$M_4 = \Delta x\left(f(c_1)+f(c_2)+f(c_3)+f(c_4)\right) \Rightarrow a = 4;\ b = 12;\ n = 4;\ \Delta x = \frac{b-a}{n} = \frac{12-4}{4} = 2$$

$$f(x) = \sqrt{x^2-1}$$

$[4, 6]$	$[6, 8]$	$[8, 10]$	$[10, 12]$
$c_1 = 5$	$c_2 = 7$	$c_3 = 9$	$c_4 = 11$
$f(c_1) = \sqrt{24}$	$f(c_2) = \sqrt{48}$	$f(c_3) = \sqrt{80}$	$f(c_4) = \sqrt{120}$

$$M_4 = 2\left(\sqrt{24}+\sqrt{48}+\sqrt{80}+\sqrt{120}\right) \approx 63.452$$
$$\int_4^{12} \sqrt{x^2-1}\,dx \approx 63.447$$

2. Use the rectangle (mid-point) rule, with n = 2, to approximate $\int_{-2}^{4} \left(x-x^2\right)dx$.

$$M_2 = \Delta x\left(f(c_1)+f(c_2)\right) \Rightarrow a = -2;\ b = 4;\ n = 2;\ \Delta x = \frac{b-a}{n} = \frac{4-(-2)}{2} = 3$$

$$f(x) = x - x^2$$

$[-2,1]$	$[1,4]$
$c_1 = -\frac{1}{2}$	$c_2 = \frac{5}{2}$
$f(c_1) = -\frac{1}{2} - \left(-\frac{1}{2}\right)^2$	$f(c_2) = \frac{5}{2} - \left(\frac{5}{2}\right)^2$
$= -\frac{1}{2} - \frac{1}{4} = -\frac{2}{4} - \frac{1}{4} = -\frac{3}{4}$	$= \frac{5}{2} - \frac{25}{4} = \frac{10}{4} - \frac{25}{4} = -\frac{15}{4}$

$$M_2 = 3\left(-\frac{3}{4} - \frac{15}{4}\right) = 3\left(-\frac{18}{4}\right) = 3\left(-\frac{9}{2}\right) = -\frac{27}{2}$$

$$M_2 = -13.5,\quad M_4 = -16.875,\quad M_8 = -17.71875,\quad M_{16} = -17.9296875$$

$$\int_{-2}^{4} \left(x - x^2\right)dx = -18$$

3. Use the rectangle (mid-point) rule, with n = 2, to approximate $\int_{-4}^{2}\left(x-2x^2\right)dx$.

$$M_2=\Delta x\left(f\left(c_1\right)+f\left(c_2\right)\right) \Rightarrow a=-4;\, b=2;\, n=2;\, \Delta x=\frac{b-a}{n}=\frac{2-(-4)}{2}=3$$

$$f(x)=x-2x^2$$

$[-4,-1]$: $c_1=-\frac{5}{2}$

$$f\left(c_1\right)=-\frac{5}{2}-2\left(-\frac{5}{2}\right)^2$$

$$=-\frac{5}{2}-2\left(\frac{25}{4}\right)=-\frac{5}{2}-\frac{25}{2}=-\frac{30}{2}=-15$$

$[-1,2]$: $c_2=\frac{1}{2}$

$$f\left(c_2\right)=\frac{1}{2}-2\left(\frac{1}{2}\right)^2$$

$$=\frac{1}{2}-2\left(\frac{1}{4}\right)=\frac{1}{2}-\frac{1}{2}=0$$

$$M_2=3\left(-15+0\right)=3\left(-15\right)=-45$$

$$M_2=-45,\quad M_4=-51.75,\quad M_8=-53.4375,\quad M_{16}=-53.859375$$

$$\int_{-4}^{2}\left(x-2x^2\right)dx=-54$$

4. Use the rectangle (mid-point) rule, with n = 3, to approximate $\int_{-2}^{1}\left(2x-3x^2\right)dx$.

$$M_3=\Delta x\left(f(c_1)+f(c_2)+f(c_3)\right) \Rightarrow a=-2;\, b=1;\, n=3;\, \Delta x=\frac{b-a}{n}=\frac{1-(-2)}{3}=1$$

$$f(x)=2x-3x^2$$

$[-2,-1]$: $c_1=-\frac{3}{2}$

$$f(c_1)=2\left(-\frac{3}{2}\right)-3\left(-\frac{3}{2}\right)^2$$

$$=-3-3\left(\frac{9}{4}\right)=-3-\frac{27}{4}=-\frac{39}{4}$$

$[-1,0]$: $c_2=-\frac{1}{2}$

$$f(c_2)=2\left(-\frac{1}{2}\right)-3\left(-\frac{1}{2}\right)^2$$

$$=-1-3\left(\frac{1}{4}\right)=-1-\frac{3}{4}=-\frac{7}{4}$$

$[0,1]$: $c_3=\frac{1}{2}$

$$f(c_3)=2\left(\frac{1}{2}\right)-3\left(\frac{1}{2}\right)^2$$

$$=1-3\left(\frac{1}{4}\right)=1-\frac{3}{4}=\frac{1}{4}$$

$$M_3=1\left(-\frac{39}{4}-\frac{7}{4}+\frac{1}{4}\right)=1\left(-\frac{45}{4}\right)=-\frac{45}{4}$$

$$M_3=-11.25,\ M_6=-11.8125,\ M_{12}=-11.953125,\ M_{24}=-11.98828125$$

$$\int_{-2}^{1}\left(2x-3x^2\right)dx=-12$$