

How far can a random walk take us?

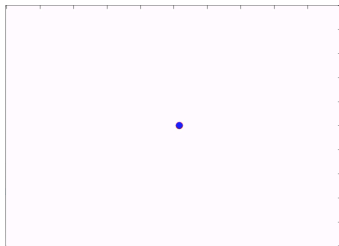
No much far!

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Undergraduate Mathematics Seminar

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# Why is there a random walk?

Nicholas Edward Brown (1849-1934)

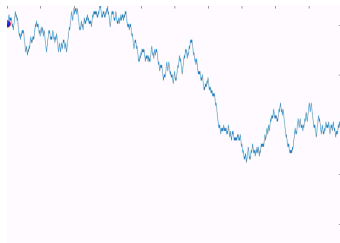


# Why is there a random walk?

Louis Bachelier 1870-1946

1900 PhD thesis under supervision of Henry Poincaré:

*Théorie de la spéculation*

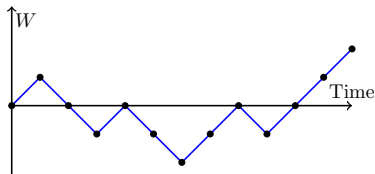
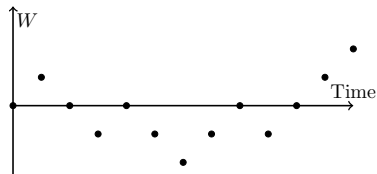


# What is a random walk?

## Coin toss probability with infinite trials

$$X_j = \begin{cases} 1 & \text{if the outcome of the } j\text{th trial is heads} \\ -1 & \text{if the outcome of the } j\text{th trial is tails} \end{cases}$$

$$S_n = S_0 + X_1 + \cdots + X_n = S_{n-1} + X_n.$$



# What is a random walk?

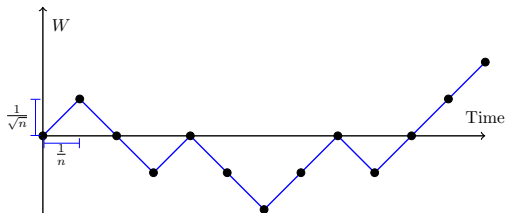
Rescale to account for small movements

$$\delta = \frac{1}{n}$$

$$X_j^\delta = \begin{cases} \sqrt{\delta} & \text{if the outcome of the } n\text{th trial is heads} \\ -\sqrt{\delta} & \text{if the outcome of the } n\text{th trial is tails} \end{cases}$$

How many time steps until  $t$ :  $n = \lfloor \frac{t}{\delta} \rfloor$

$$S_t = S_0 + X_1 + \cdots + X_n = S_{n-1} + X_n.$$



Thank you!

Q& A!