A Riemannian BFGS Method for Nonconvex Optimization Problems

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In the Euclidean setting, the Broyden–Fletcher–Goldfarb–Shanno (BFGS) method has been viewed as the best quasi-Newton method for solving unconstrained optimization problems. Its global and superlinear local convergence has been analyzed by many researchers for convex problems. In contrast, for nonconvex problems, little is known about the convergence of the plain BFGS method. This prompted efforts to modify the BFGS method such that the global convergence is guaranteed. Recently, an example of nonconvex problem in [Dai13] is given and shows that the BFGS method is unable to converge, which implies that modifications are necessary if the global convergence is desired for general problems.

In the Riemannian setting, several Riemannian versions of BFGS method have appeared and only two of them [RW12, HGA14] have general discussions with complete global and local convergence analyses rather than only specific to a cost function or a manifold. In spite of this, both of them require the cost functions to satisfy a Riemannian version of convexity in their analyses. In this presentation, we consider to update the Hessian approximation cautiously and combine it with a weak line search condition. The global and local superlinear convergence analyses are given and the joint diagonalization problem is chosen as an example to demonstrate numerical performance.

The Riemannian BFGS method in this presentation has another advantage over the approaches in [RW12, HGA14]. The Riemannian versions of BFGS method in [RW12, HGA14] and here all rely on the notion of retraction and vector transport. Both of Riemannian BFGS methods involve some information about the vector transport of differentiated retraction which may be not available to users or too expensive. In contrast, the Riemannian version of BFGS method in this presentation does not require the usage of differentiated retraction.

References

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