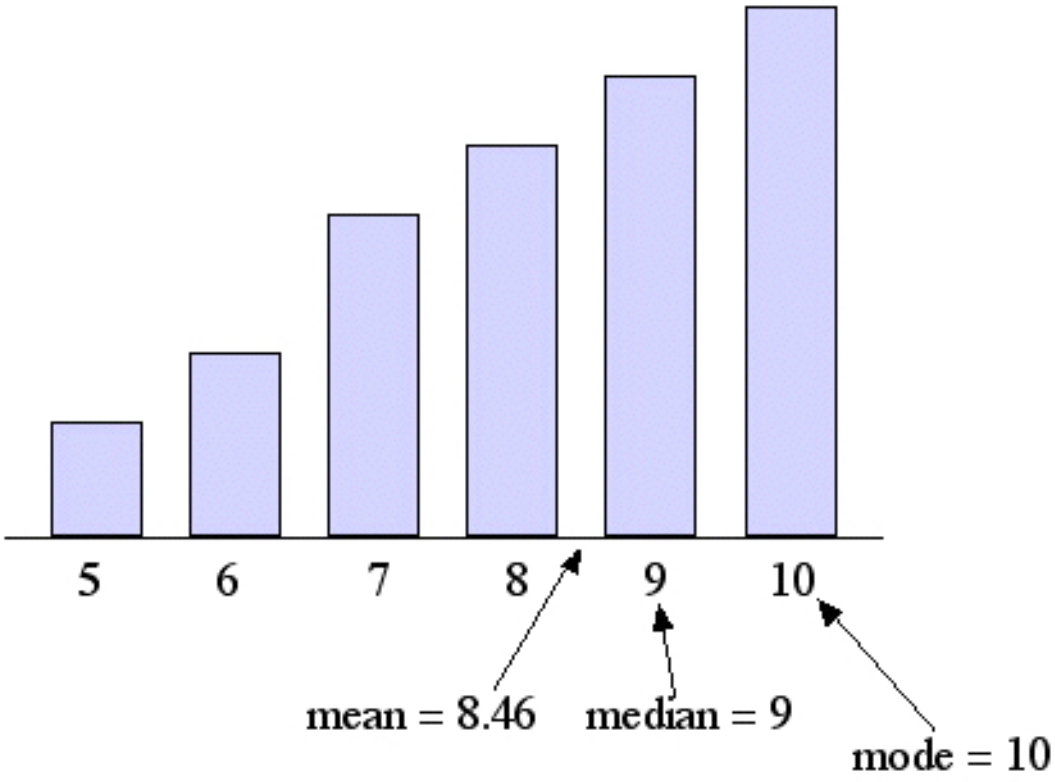
UNIT 2 MODULE 10

RELATIONSHIPS BETWEEN MEAN, MEDIAN and MODE in SPECIAL DISTRIBUTIONS

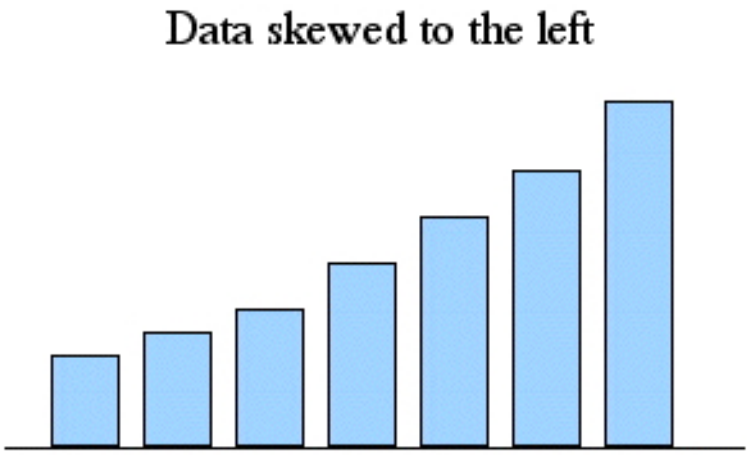
EXAMPLE 2.10.1
Refer to EXAMPLE 2.9.6.
Make a bar graph (using vertical bars) for the data in that example. On the horizontal axis, make note of the positions of the mean, median and mode.

EXAMPLE 2.10.1 SOLUTION
The bar graph looks like this:



DATA SKEWED TO THE LEFT

The bar graph above is an illustration of a special kind of data distribution. The distribution has the property that as the values increase the frequencies increase as well. This means that on the bar graph, the columns get taller as we look from left to right. In such a distribution, we say that the data is **skewed to the left.**



In EXAMPLE 2.10.1 we saw that for a specific distribution that was skewed to the left, the mode (10) was the greatest of the three measures of central tendency, the mean (8.46) was the least of the three measures of central tendency, and the median was in between. This illustrates a typical property of data that is skewed to the left.

TYPICAL PROPERTY OF DATA SKEWED TO THE LEFT

Of the three measures of central tendency, the mode will be the greatest, the mean will be the least, and the median will be in between.

This relationship is summarized as follows:
mode > *median* > *mean*

which is the same as

mean < *median* < *mode*

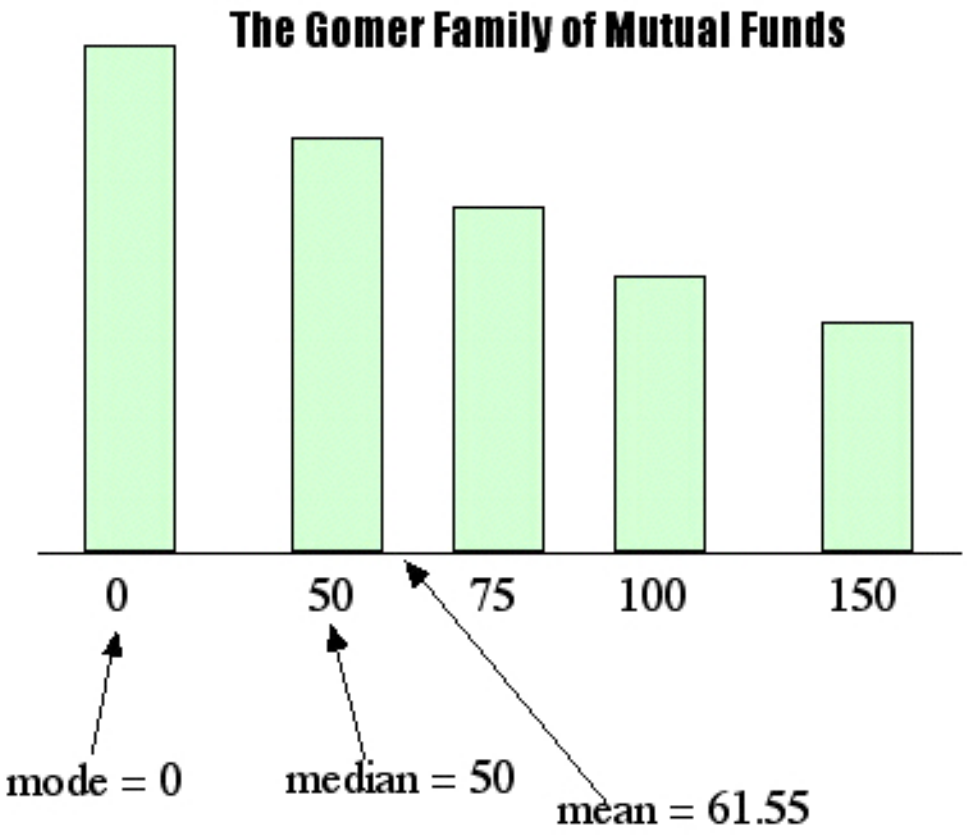
(Note: there may be exceptions to this trend.)

EXAMPLE 2.10.2

Refer to the data in EXAMPLE 2.9.9. Make a bar graph that displays the data. Use vertical bars. Additionally, note the relative positions of the mean, median and mode on the horizontal axis.

EXAMPLE 2.10.2 SOLUTION

The bar graph looks like this:



DATA SKEWED TO THE RIGHT

The data in the previous example had the following property: in every case, increasing values are associated with decreasing frequencies. This corresponds to the fact that on the bar graph, the bars get shorter as we look from left to right. A distribution that has those properties is called **skewed to the right**.

In EXAMPLE 2.10.2 we saw that for data skewed to the right, the three measures of central tendency had this numerical relationship: the mode (0) was the least, the mean (61.55) was the greatest, and the median was in between. This illustrates the following typical property of data skewed to the right:

TYPICAL PROPERTY OF DATA SKEWED TO THE RIGHT

Of the three measures of central tendency, the mean will be the greatest, the mode will be the least, and the median will be in between.

This relationship is summarized as follows:

mean > *median* > *mode*

which is the same as

mode < *median* < *mean*

(Note: there may be exceptions to this trend.)

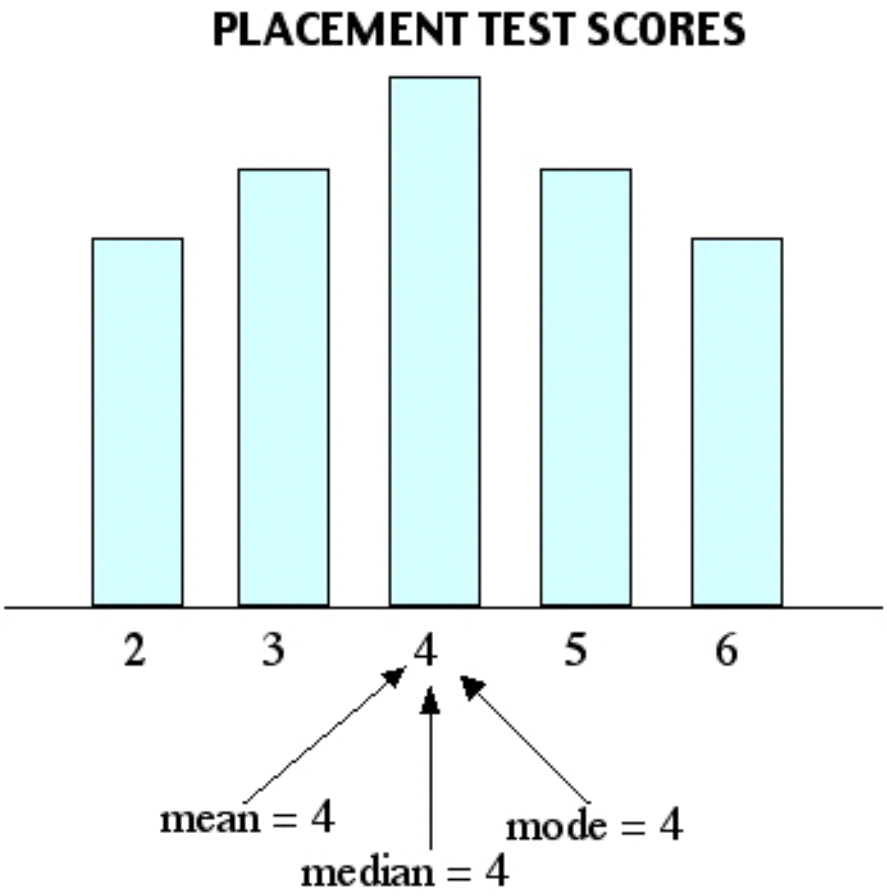
EXAMPLE 2.10.3

The distribution of scores on an aptitude test is given below. Find the mean, median and mode. Make a bar graph for the distribution, using vertical bars. Show the relative positions of the three measures of central tendency on the horizontal axis.

Value	Frequency
2	24
3	30
4	36
5	30
6	24

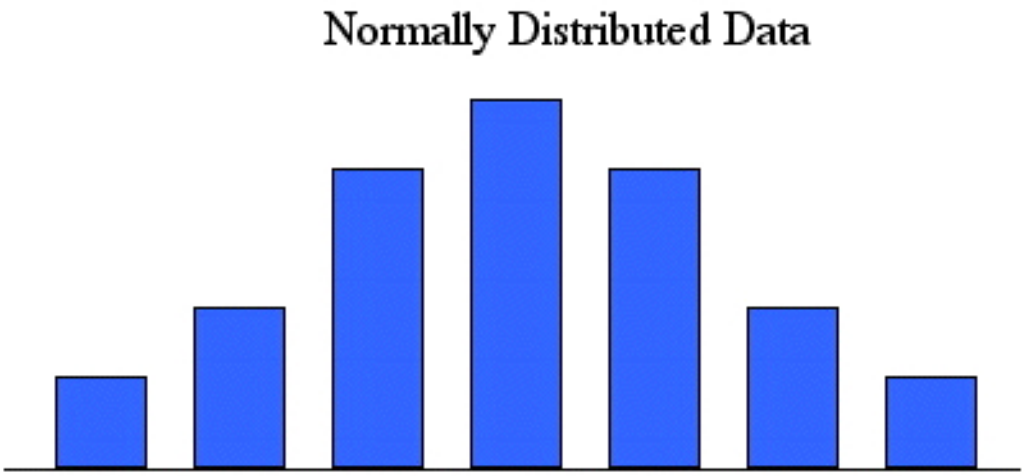
EXAMPLE 2.10.3 SOLUTION

Using the methods from Unit 2, Module 9 you will find that the mean, median and mode are all equal to 4. The bar graph looks like this:



NORMALLY DISTRIBUTED DATA

In the previous example we saw a distribution whose bar graph was symmetric, with the highest bar in the middle. This kind of distribution is referred to as **normal**, or **normally distributed** data. When graphed, normally distributed data will typically have an appearance similar to this:



TYPICAL PROPERTY OF NORMALLY DISTRIBUTED DATA

When data is normally distributed, the mean, median and mode will all be the same:

mean = median = mode

EXAMPLE 2.10.4

The graph below shows the distribution according to height of a group of jockeys at a south Florida horse track. Select the statement that correctly describes a relationship between measures of central tendency for this distribution.

A. The mean is less than the mode.
B. The mode and the mean are the same.
C. The median is greater than the mode.
D. The median and the mean are the same.

EXAMPLE 2.10.4 solution

First, we note that there is not sufficient information for us to compute the mean or the median, since we are not provided with frequencies for this distribution.

One way to solve this problem is by referring to information in a previous example. We note that the data is skewed to the right. According to EXAMPLE 2.10.2 above, when data is skewed to the right the following relationship is typical:
mean > median > mode.
This allows us to recognize that choice C is correct.

There is another way to recognize that choice C is correct, without having to memorize the facts from the previous examples.

First, we observe that although we cannot precisely compute the mean or the median, there is no doubt as to the value of the mode: the mode is 58 (the value under the tallest column).

We also see that not only is 58 the mode, but 58 is also the smallest value in the distribution (the values range from 58 to 62). This allows us to instantly state the relationship between the mode and the mean: *the mean must be greater than the mode* (because in this case the mode is also the smallest value, and in any collection of numbers that aren't all identical, the average value must be greater than the smallest number).

Mean > mode

Similar reasoning allows us to observe that the *median must also be greater than the mode.* (Note: if the tallest column were taller than the heights of all the other columns combined, then the median and mode would be equal).

Median > mode

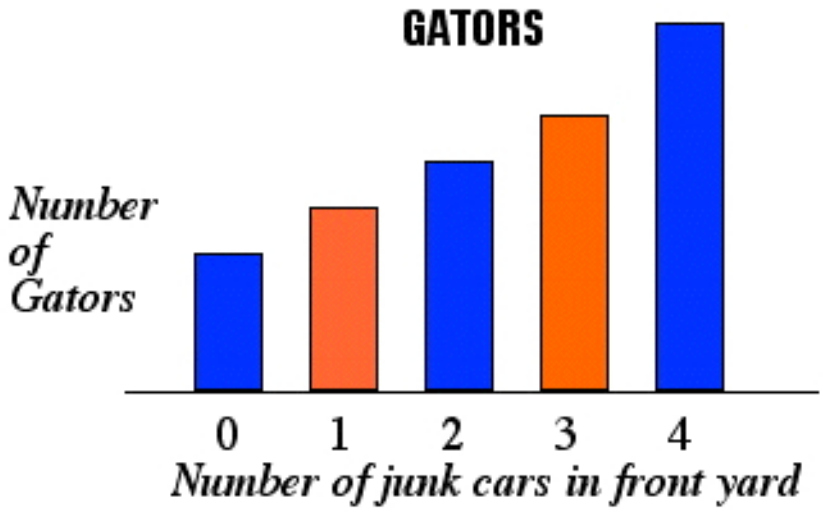
What about the relationship between the mean and the median? We state the following rule of thumb: **in a skewed distribution, it is reasonable to assume that the median will fall between the mean and the mode.** As in earlier observations, we stipulate that there are occasional exceptions to this rule of thumb.

Mean > median > mode

We see that choice C is the best choice.

EXAMPLE 2.10.5

The graph below shows the distribution of a group of Gator fans according the number junk cars in their back yards. Select the statement that correctly gives a relationship between measures of central tendency for this distribution.



A. The mean is the same as the median.
B. The mean is less than the mode.
C. The mode is less than the mean.
D. The median is less than the mean.

EXAMPLE 2.10.6

Euclid is soliciting donations to fund his spring break trip to Cancun. More than half of his donations have been in the amount of 50¢. The others have been equally divided among values of 25¢ and values of 75¢. Select the statement that correctly gives a relationship between measures of central tendency for this distribution.

A. The median is greater than the mean.

B. The mean is less than the mode.

C. The mean is equal to the median.

D. The mode is less than then median.

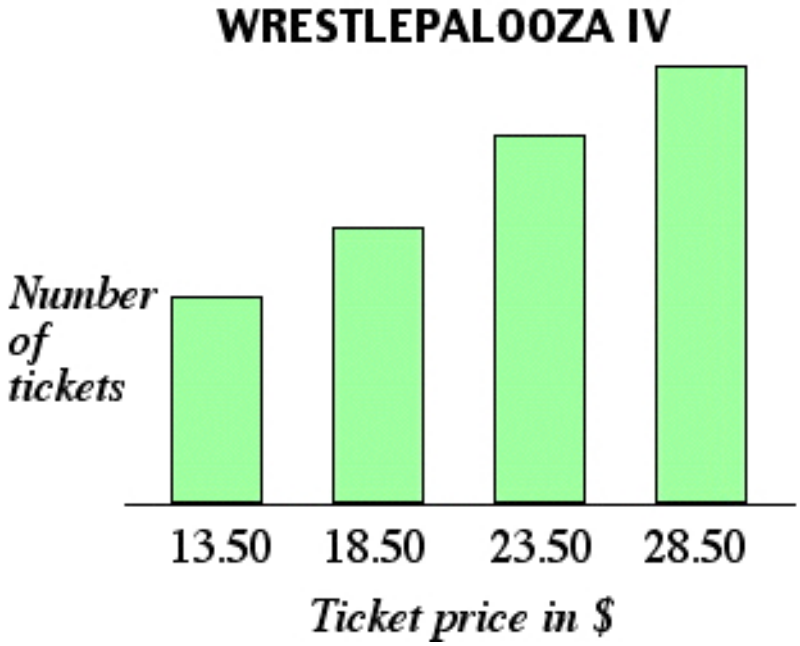
EXAMPLE 2.10.7

Whenever Gomer goes for a stroll, he waves at every person who passes by. He finds that the greatest number of people (nearly half of the people) do not wave back (that is, they wave back with *zero* fingers). Of the others, most wave back with one finger, while some wave back with five fingers. Select the statement that is true regarding this distribution of fingers.

A. The mean is less than the mode.
B. The median is less than the mean.
C. The median is less than the mode.
D. The mean is the same as the median.

EXAMPLE 2.10.8

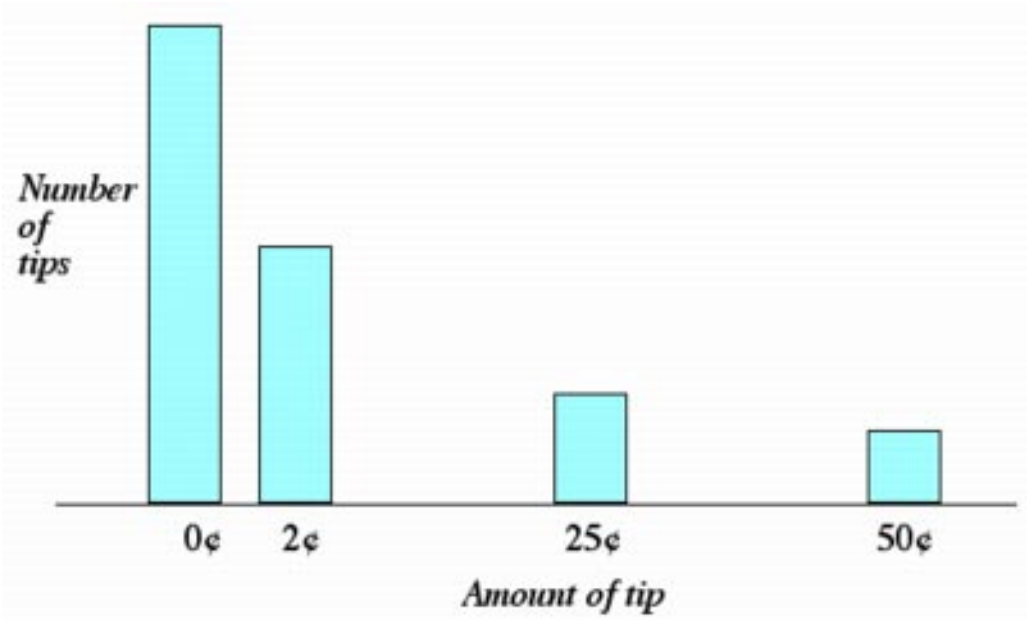
The graph below shows the distribution according to price of tickets sold for Wrestlepalooza IV. Select the statement that correctly describes a relationship between measures of central tendency.



A. The mean is less than the mode.
B. The median is less than the mean.
C. The mode is the same as the mean.
D. The mean is the same as the median.

EXAMPLE 2.10.9

Gomer works as a waiter. The graph below shows the distribution of his tips last week. Select the statement that correctly describes a relationship between measures of central tendency for this distribution.



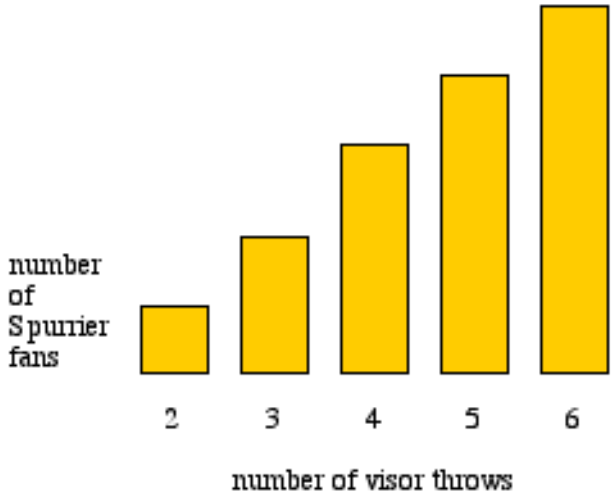
A. The median is greater than the mean.
B. The median is greater than the mode.
C. The mode is the same as the median.
D. The mean is less than the mode.

WORLD WIDE WEB NOTE

For more practice on problems like these, visit the companion website and try THE CENTRAL TENDERIZER PART 2.

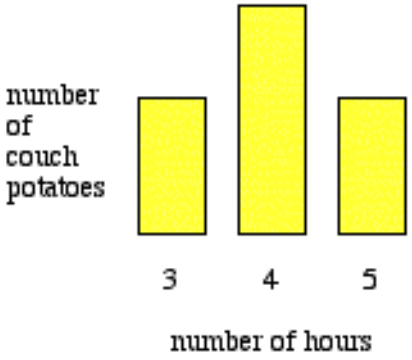
PRACTICE EXERCISES

1. A number of Spurrier fans were asked 'How many times will Coach throw his visor next week?' The distribution is summarized in the bar graph below. Select the statement that correctly describes a relationship between measures of central tendency for this distribution.



A. The mean is greater than the median.
B. The median is equal to the mode.
C. The mode is greater than the mean.
D. The mode is less than the mean.

2. A number of couch potatoes were asked 'How many hours of Nintendo did you play yesterday?' The distribution is summarized in the bar graph below. Select the statement that correctly describes a relationship between measures of central tendency for this distribution.



A. The median is less than the mean.
B. The mode is greater than the mean.
C. The mode is equal to the median.
D. The mean is greater than the mode.

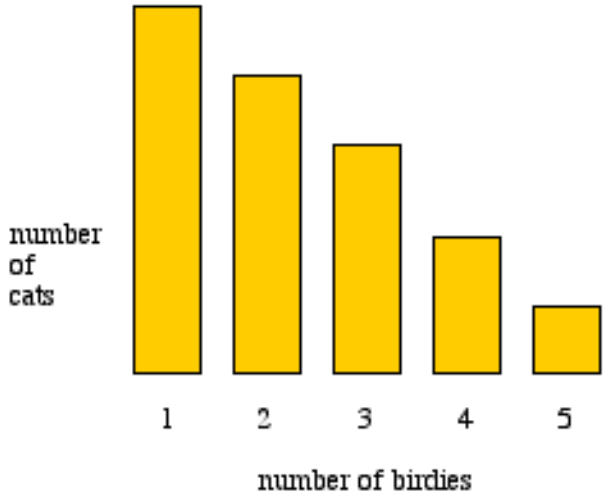
3. A number of toads were asked 'How many warts do you have?' The greatest number (not quite half) responded '7'. Among the others, most replied '5' and a few replied '3'. Select the statement that correctly describes a relation between measures of central tendency for this distribution.

A. The mode is greater than the median. B. The mode is equal to the median.
C. The mode is less than the median. D. The mode is less than the mean.

4. A number of dogs were asked 'How many garbage cans did you tip over last week?' . Most (approximately half) responded '4'. The others were equally divided among those who replied '3' and those who replied '5'. Select the statement that correctly describes a relation between measures of central tendency for this distribution.

A. The mean is equal to the median. B. The median is less than the mode.
C. The mode is less than the median. D. The mode is less than the mean.

5. A number of cats were asked 'How many birdies did you eat last month?' The distribution is summarized in the bar graph below. Select the statement that correctly describes a relationship between measures of central tendency for this distribution.



A. The mode is greater than the median. B. The mode is greater than the mean.
C. The mode is equal to the mean. D. The mean is greater than the mode.

6. A number of Gen X-ers were asked 'How many hours did you spend slacking yesterday?' The greatest number (not quite half) responded '2'. Among the others, most replied '3' and a few replied '4'. Select the statement that correctly describes a relation between measures of central tendency for this distribution.

A. The mean is equal to the median. B. The mode is greater than the median.
C. The median is greater than the mode. D. The mode is greater than the mean.

ANSWERS TO LINKED EXAMPLES

EXAMPLE 2.10.5 B
EXAMPLE 2.10.6 C
EXAMPLE 2.10.7 B
EXAMPLE 2.10.8 A
EXAMPLE 2.10.9 B

ANSWERS TO PRACTICE EXERCISES

1. C **2.** C **3.** A **4.** A **5.** D **6.** C