

UNIT 2 MODULE 2
ANALYZING PREMISES, FORMING CONCLUSIONS

First, we define a **Trivial Valid Conclusion**

No matter how poorly formulated an argument may be, it is always possible to form a valid conclusion by merely restating one of the premises and calling it the conclusion. Such a conclusion is called a **trivial valid conclusion**.

EXAMPLES OF ARGUMENTS HAVING TRIVIAL VALID CONCLUSIONS

1. It is raining.

Therefore, it is raining.

2. My feet hurt and I'm having a bad hair day.

Therefore, my feet hurt.

3. If I work hard, then I will succeed.

I succeeded.

Therefore, I succeeded.

In the work that follows, we will analyze collections of premises in order to recognize whether valid conclusions are possible. In all cases, *we will exclude trivial conclusions*.

UNIT 2 MODULE 2

We begin this module with a summary of important results observed in Unit 2, Module 1.

SUMMARY: SOME COMMON PATTERNS OF *VALID* REASONING

DIRECT REASONING

$$\begin{array}{l} p \rightarrow q \\ \underline{p} \\ \therefore q \end{array}$$

CONTRAPOSITIVE REASONING

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

DISJUNCTIVE SYLLOGISMS

$$\begin{array}{ll} p \vee q & p \vee q \\ \underline{\sim q} & \underline{\sim p} \\ \therefore p & \therefore q \end{array}$$

TRANSITIVE REASONING

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

SUMMARY: SOME COMMON PATTERNS OF *INVALID* REASONING

FALLACY OF THE CONVERSE

$$\begin{array}{l} p \rightarrow q \\ \underline{q} \\ \therefore p \end{array}$$

FALLACY OF THE INVERSE

$$\begin{array}{l} p \rightarrow q \\ \underline{\sim p} \\ \therefore \sim q \end{array}$$

DISJUNCTIVE FALLACIES

$$\begin{array}{ll} p \vee q & p \vee q \\ \underline{p} & \underline{q} \\ \therefore \sim q & \therefore \sim p \end{array}$$

FALSE CHAINS

$$\begin{array}{ll} p \rightarrow q & p \rightarrow q \\ \underline{p \rightarrow r} & \underline{r \rightarrow q} \\ \therefore q \rightarrow r & \therefore p \rightarrow r \end{array}$$

HACKING MATHEMATICS

These patterns of reasoning are especially useful for exercises like the following example:

EXAMPLE 2.2.1

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

I use my computer or I don't get anything done.

I get something done.

A. I use my computer.

B. I don't use my computer.

C. I use an abacus.

D. None of these is warranted.

EXAMPLE 2.2.1 Solution

This argument problem differs from the earlier examples in *that we aren't given a conclusion for the argument*. This means that if we try to use a truth table to analyze the argument, we may not be sure what statement to use in the "conclusion" column.

However, it is easily solved by reference to the patterns of reasoning summarized above.

We symbolize the premises:

Let p be the statement "I use my computer." Let q be the statement "I don't get anything done."

Then the symbolic representation for the two premises has this form:

$$p \vee q$$

$$\sim q$$

Now we observe that this is the premise arrangement for one form of **Disjunctive Syllogism**, which is a form of **valid** reasoning. The pattern tells us that we can form a non-trivial valid conclusion:

$$p \vee q$$

$$\underline{\sim q}$$

$$\therefore p$$

That is, when "Therefore, p " is attached as the conclusion we will have a valid argument.

This means that a valid conclusion is warranted, namely "I use my computer," which is choice A.

EXAMPLE 2.2.2

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If I win the Lotto, then I'll reform my life. I reformed my life.

- A. I didn't win the Lotto.
- B. I'll run for President as the Reform Party nominee.
- C. I won the Lotto.
- D. None of these is warranted.

EXAMPLE 2.2.2 solution

Let p be the statement "I win the Lotto." Let q be the statement "I reform my life." The premise arrangement has this symbolic form:

$$p \rightarrow q$$

$$q$$

We recognize that this is the premise arrangement for an invalid argument (Fallacy of the Converse). This tells us that the best-sounding choice ("C. I won the Lotto") is not correct, because that choice would result in an invalid argument.

More importantly, *because we have the premise arrangement for an invalid argument, it is not possible to produce a non-trivial valid conclusion.* This tells us that the correct choice must be D.

EXAMPLE 2.2.3

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If we strive, then we excel. We didn't strive.

- A. We excelled.
- B. We didn't excel.
- C. We didn't inhale.
- D. None of these is warranted.

EXAMPLE 2.2.4

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If we win, then we celebrate. We aren't celebrating.

- A. We won.
- B. We didn't win.
- C. We stink.
- D. None of these is warranted.

EXAMPLE 2.2.5

Given:

i. If my car doesn't start, then I'll be late for work; and

ii. I'm not late for work.

select the statement that is a valid conclusion, if a valid conclusion is warranted.

- A. My car started.
- B. I rode the bus.
- C. I'm late for work.
- D. None of these is warranted.

EXAMPLE 2.2.6

Given:

i. All nurses are kind; and

ii. Florence isn't a nurse.

select the statement that is a valid conclusion, if a valid conclusion is warranted.

- A. Florence is a city in Italy.
- B. Florence isn't kind.
- C. Florence is kind.
- D. None of these is warranted.

EXAMPLE 2.2.7

Given:

i. No kittens are fierce; and

ii. Fluffy isn't fierce.

select the statement that is a valid conclusion, if a valid conclusion is warranted.

- A. Fluffy is a kitten.
- B. Fluffy has fleas.
- C. Fluffy isn't a kitten.
- D. None of these is warranted.

WORLD WIDE WEB NOTE

For practice on problems like these, visit the companion website and try THE DEDUCER.

SPECIAL CASES INVOLVING TRANSITIVE REASONING

EXAMPLE 2.2.8

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If you want a better grade, then you bring an apple for the teacher.

If you bring an apple for the teacher, then you expose the teacher to dangerous agricultural chemicals.

A. If you expose the teacher to dangerous agricultural chemicals, then you want a better grade.

B. If you don't expose the teacher to dangerous agricultural chemicals, then you don't want a better grade.

C. You want a better grade.

D. None of these is warranted.

EXAMPLE 2.2.8 solution

Let p be the statement "You want a better grade."

Let q be the statement "You bring an apple for the teacher."

Let r be the statement "You expose the teacher to dangerous agricultural chemicals."

The premise arrangement has this form:

$$p \rightarrow q$$

$$q \rightarrow r$$

We see that this is the arrangement of premises for **Transitive Reasoning**, which is a form of **valid** reasoning. This means that we will be able to form a valid conclusion, namely:

$$p \rightarrow q$$

$$\underline{q \rightarrow r}$$

$$\therefore p \rightarrow r$$

In words, the valid conclusion is "If you want a better grade, then you expose the teacher to dangerous agricultural chemicals."

Unfortunately, this isn't one of the listed choices. We may now refer to the following fundamental fact:

If we have a statement that is a valid conclusion for an argument, then any *equivalent* statement will also be a valid conclusion.

In this particular case, the statement $p \rightarrow r$ is a valid conclusion, so its equivalent **contrapositive** $\sim r \rightarrow \sim p$ will also be a valid conclusion. In words, the **contrapositive** of "If you want a better grade, then you expose the teacher to dangerous agricultural chemicals," will also be a valid conclusion. This is the statement "If you don't expose the teacher to dangerous agricultural chemicals, then you don't want a better grade."

The correct choice is B.

EXAMPLE 2.2.9

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

All people who get many tickets are uninsurable.

All careless drivers get many tickets.

All people who are uninsurable have bad credit ratings.

- A. All careless drivers have bad credit ratings.
- B. If your car is repossessed because you have bad credit, then you are a car-less driver.
- C. All people are uninsurable get many tickets.
- D. None of these is warranted.

The previous two examples illustrate the following procedures that can be employed in order to use **Transitive Reasoning** to form a conclusion.

TO FORM A VALID CONCLUSION USING TRANSITIVE REASONING:

1. We may replace any premises, or the conclusion, with equivalent statements. In particular, conditional statements may be replaced with their **contrapositives** (but not with converses or inverses).

2. We may rearrange the order in which the premises are listed. In particular, in order to use Transitive Reasoning, we will rearrange them premises so that **the antecedent of the first premise is a variable that appears only one time in the entire premise scheme.** We will then continue rearranging the order of the premises, and perhaps replacing premises with equivalent statements, so that **the antecedent of each premise is exactly the same as the consequent of the preceding premise.**

If at any point it is impossible to continue this linkage of premises, then the argument involves a false chain, and so it is not possible to form a valid conclusion that uses every premise (although it may be possible to form valid conclusions that use only a subset of the original set of premises).

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HACKING MATHEMATICS

EXAMPLE 2.2.10

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If I win the Lotto, then I won't need a job.

If I have lots of bills, then I will need a job.

A. If I have lots of bills, then I didn't win the Lotto.

B. If I didn't win then Lotto, then I have lots of bills.

C. If I don't need a job, then I won the Lotto.

D. A valid conclusion is not warranted.

EXAMPLE 2.2.11

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If I invest wisely, then I won't lose my money.

If I don't invest wisely, then I buy junk bonds.

If I read *Investor's Weekly*, then I won't buy junk bonds.

A. If I invest wisely, then I read *Investor's Weekly*.

B. If I buy junk bonds, then I don't invest wisely.

C. If I lose my money, then I don't read *Investor's Weekly*.

D. If I eat junk food, then I invest weakly.

E. None of these is warranted.

EXAMPLE 2.2.12

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

All poodles love noodles. All pooches love noodles.

A. All pooches are poodles.

B. All poodles are pooches.

C. No poodles are pooches.

D. None of these is warranted.

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EXAMPLE 2.2.13

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

All compact cars are uncomfortable.

All compact cars get good gas mileage.

- A. No compact cars get good gas mileage.
- B. All compact cars are cheap.
- C. All cars that get good gas mileage are uncomfortable.
- D. None of these is warranted.

EXAMPLE 2.2.14

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

People who dislike cats are degenerates. All pirates own parrots. People who like cats never own parrots.

- A. If you are a pirate, then you are a degenerate.
- B. All degenerates own parrots.
- C. All cats lick parrots.
- D. None of these is warranted.

EXAMPLE 2.2.15

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

No body builders are weak.

All professional wrestlers are body builders.

All 300-pound men with bleached hair and sequined tights are professional wrestlers.

Plato is weak.

- A. Plato is a 300-pound man with bleached hair and sequined tights.
- B. Plato is not a 300-pound man with bleached hair and sequined tights.
- C. Plato is the Masked Warrior and hits people with chairs.
- D. None of these is a valid conclusion.

HACKING MATHEMATICS

EXAMPLE 2.2.16

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

Sylvester isn't a parakeet. Elephants never squawk.
All parakeets squawk. No elephants are tiny.

- A. Sylvester is an elephant.
- B. Sylvester isn't tiny.
- C. All parakeets are tiny.
- D. None of these is warranted.

EXAMPLE 2.2.17

Select the statement that is a valid conclusion from the following premises, if a valid conclusion is warranted.

If you aren't a good stirrer, then you aren't handy with a swizzle stick.
If you are a graduate of *Billy Bob's Big Bold School of Mixology*, then you are a bartender.
No good stirrers have weak wrist muscles.
If you don't have weak wrist muscles, then you have a firm handshake.
All bartenders are handy with a swizzle stick.

- A. If you are a graduate of *Billy Bob's Big Bold School of Mixology*, then you don't have a firm handshake.
- B. If you don't have a firm handshake, then you aren't a graduate of *Billy Bob's Big Bold School of Mixology*.
- C. If you have a firm handshake, then you are a graduate of *Billy Bob's Big Bold School of Mixology*.
- D. None of these is warranted.

UNIT 2 MODULE 2

PRACTICE EXERCISES

1 – 21: Select the statement that is a valid conclusion from the following premises, if a non-trivial valid conclusion is warranted. If none of the given conclusions is warranted, select the option that indicates so.

1. If you pet that wolverine, he will tear off your fingers. You are not missing any fingers.
A. You petted that wolverine. B. You didn't pet that wolverine.
C. Check your toes. D. None of these is warranted.

2. All carpenters are patient. Gomer isn't a carpenter.
A. Gomer isn't patient. B. Some carpenters aren't patient.
C. Gomer is a painter. D. None of these is warranted.

3. I won't go to school or I will live off campus. I'm going to school.
A. I'm living off campus. B. I'm living on campus.
C. I'd rather live in a tent than on campus. D. None of these is warranted.

4. If we go to the concert, you will drive. You are driving.
A. We are going to the concert.
B. We aren't going to the concert.
C. If we don't go to the concert, you won't drive.
D. None of these is warranted.

5. If you park in the wrong spot, you'll get a parking ticket. You didn't park in the wrong spot.
A. You didn't get a parking ticket.
B. You got a parking ticket.
C. The attendant gave you a ticket because your car is ugly.
D. None of these is warranted.

6. All misers are skinflints. All skinflints are cheapskates. Cheapskates never donate to Public Broadcasting pledge drives. Gomer donates to public broadcasting pledge drives.
A. Some cheapskates aren't misers. B. Gomer isn't a miser.
C. Gomer dusts his furniture with Pledge. D. None of these is warranted.

7. All school-crossing guards are cautious. Socrates isn't cautious.
A. Socrates was a Greek philosopher. B. Socrates is dead.
C. Socrates isn't a school-crossing guard. D. None of these is warranted.

8. All conspiracy buffs are suspicious by nature. Aristotle is suspicious by nature.
A. Aristotle is a conspiracy buff. B. Aristotle isn't a conspiracy buff.
C. Aristotle conspires in the buff. D. None of these is warranted.

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17. (Adapted from Lewis Carroll) No promise-breakers are trustworthy. All wine-drinkers are communicative. If you aren't a promise-breaker then you are honest. All pawnbrokers are wine-drinkers. Communicative people are trustworthy.

- A. All pawnbrokers are untrustworthy.
- B. If you aren't honest then you aren't a pawnbroker.
- C. If you aren't a promise-breaker then you are communicative.
- D. None of these is warranted.

18. If I sell some of my books then I'll pay the rent. I didn't pay the rent.

- A. Some of my books are sold.
- B. All of my books are sold.
- C. None of my books is sold.
- D. None of these is warranted.

19. If you wear dark glasses then you look mysterious. If you wear dark socks with sandals then you look dorky. If you look dorky then you don't look mysterious.

- A. If you wear dark socks with sandals then you don't wear dark glasses..
- B. If you look dorky then you wear dark glasses.
- C. If you don't wear dark glasses then you wear dark socks with sandals.
- D. None of these is warranted.

20. If your house has termites, then you can't sell it. If your house has passed inspection then you can sell it.

- A. If you can sell your house, then it has passed inspection.
- B. If your house doesn't have termites then it has passed inspection.
- C. If your house has passed inspection then it doesn't have termites.
- D. None of these is warranted.

21. All of my classes are cancelled or some of my plans are cancelled. None of my plans is cancelled.

- A. Some of my classes aren't cancelled.
- B. None of my classes is cancelled.
- C. All of my classes are cancelled.
- D. None of these is warranted.

HACKING MATHEMATICS

ANSWERS TO LINKED EXAMPLES

EXAMPLE 2.2.3	D
EXAMPLE 2.2.4	B
EXAMPLE 2.2.5	A
EXAMPLE 2.2.6	D
EXAMPLE 2.2.7	D
EXAMPLE 2.2.9	A
EXAMPLE 2.2.10	A
EXAMPLE 2.2.11	C
EXAMPLE 2.2.12	D
EXAMPLE 2.2.13	D
EXAMPLE 2.2.14	A
EXAMPLE 2.2.15	B
EXAMPLE 2.2.16	D
EXAMPLE 2.2.17	B

ANSWERS TO PRACTICE EXERCISES

- | | |
|---|---|
| 1. B (contrapositive reasoning) | 2. D (fallacy of inverse) |
| 3. A (disjunctive syllogism) | 4. D (fallacy of converse) |
| 5. D (fallacy of inverse) | 6. B (transitive reasoning/direct reasoning) |
| 7. C (contrapositive reasoning) | 8. D (fallacy of converse) |
| 9. D (false chain) | 10. A (transitive reasoning) |
| 11. A (direct reasoning) | 12. C (contrapositive reasoning) |
| 13. A (transitive or direct reasoning) | 14. D (false chain) |
| 15. B (transitive reasoning) | 16. C (transitive reasoning) |
| 17. B (transitive reasoning) | 18. C (contrapositive reasoning) |
| 19. A (transitive reasoning) | 20. C (transitive reasoning) |
| 21. C (disjunctive syllogism) | |