UNIT 2 MODULE 3

CATEGORICAL SYLLOGISMS AND DIAGRAMMING

Consider the following argument:

Some lawyers are judges. Some judges are politicians. Therefore, some lawyers are politicians.

Although the premises and conclusion of this argument sound reasonable, and although the structure of the argument looks similar to transitive reasoning, this argument is **invalid**.

In order to show that the argument is invalid, all we have to do is conceive of a situation in which the conclusion is false, while both premises are true. In order to do so, it helps if we imagine a world with a small population of lawyers, judges and politicians. Suppose there are only two lawyers, Alice and Bill, and that Bill is also a judge, but Alice isn't. Suppose that in addition to Bill there is only one other judge, Carla, and Carla is also a politician, but Bill isn't a politician. Finally, suppose there is one other politician, Don, who isn't a lawyer and isn't a politician. In this conceivable world, some lawyers are judges (Bill), and some judges are politicians (Carla), but no lawyers are politicians. Since it is possible to conceive of a situation in which the conclusion is false while both premises are true, this argument is invalid.

The previous argument is an example of a **CATEGORICAL SYLLOGISM**, which is an argument involving two premises, both of which are **categorical statements**. Categorical statements are statements of the form "all are...," "none are..." or "some are..." A categorical statement of the form "all are..." is also called a **positive universal** statement. A categorical statement of the form "none are..." is also called a **negative universal** statement. A categorical statement of the form "some are..." or "some aren't is also called an **existential statement.**

In this discussion we are primarily concerned with categorical syllogisms in which at least one premise is an existential statement, because such arguments cannot be analyzed using the methods of Unit 2 Module 1.

Existential statements

A statement of the form "Some are...," such as "Some lawyers are judges," is conceptually quite different from a universal statement, in that it cannot be restated in terms of logical connectives in any way that is of practical use. Whereas a positive universal statement such as "All cats are mammals" can be informally restated as "If __ is a cat, then __ is a mammal," and whereas a negative universal statement such as "No cats are dogs" can be restated as "If __ is a cat, then __ isn't a dog," it is not possible to make such a transition with an existential statement such as "Some mammals are predators."

This means that the techniques of Unit 2 Module 1, which are based on truth tables and logical connectives, are of no use for arguments involving the existential statement.

Diagramming categorical statements

(Note: a discussion of diagramming categorical statements was included at the end of Unit 1 Module 4, as well).

There is an extensive literature on the topic of categorical syllogisms, dating back to medieval scholarship and earlier. This includes an impressive body of special terminology, symbols, and characterizations of forms, which a student might encounter in a more intense study of the subject, such as in a history of philosophy course.
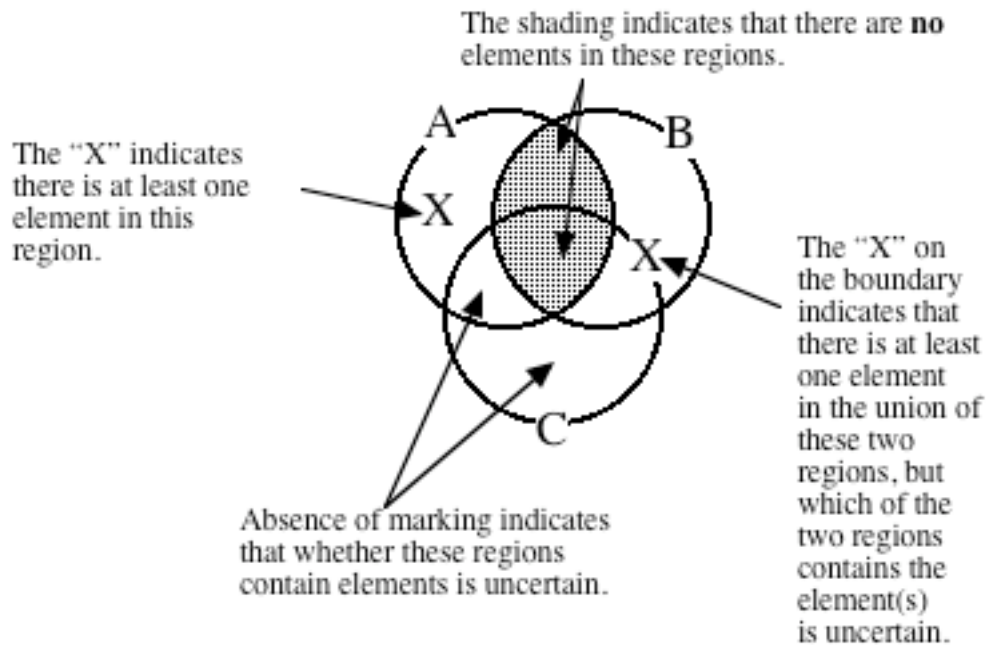
This discussion will be limited to the presentation of a method of analyzing categorical syllogisms through the use of three-circle Venn diagrams. This method is called **diagramming**.

Individual statements are diagrammed as follows.

1. Use shading to diagram universal statements, by shading out any region that is known to contain no elements.

2. Use an "X" to diagram an existential statement. If a region is known to contain at least one element, place an "X" in that region. If it is uncertain which of two regions must contain the element(s), then place the "X" on the boundary between those two regions.

3. If a region contains no marking, then it is uncertain whether or not that region

contains any elements.

The marked Venn diagram below illustrates these ideas.

The shading indicates that there are **no** elements in these regions.

The "X" indicates there is at least one element in this region.

The "X" on the boundary indicates that there is at least one element in the union of these two regions, but which of the two regions contains the element(s) is uncertain.

Absence of marking indicates that whether these regions contain elements is uncertain.

Diagramming categorical syllogisms

To test the validity of a categorical syllogism, follow these steps.

1. In order to be valid, a categorical syllogism must have at least one premise that is a universal statement. If none of the premises is a universal statement, then the argument is invalid, and we are done. The following steps assume that at least one premise is a universal statement.

2. Begin by diagramming the universal premise(s). A universal statement will have the effect of shading (blotting out, so to speak) some region of the diagram, because a universal statement will always assert, directly or otherwise, that some region of the diagram has no elements.

3. Confining your attention to the part of the diagram that is **un**shaded, diagram an existential premise by placing an "X" in a region of the diagram that is known to contain at least one element. If it is uncertain which if two regions should contain the element(s), place the "X" on the boundary between those two regions.

4. After diagramming the premises, if the diagram shows the conclusion of the argument to be true, then the argument is valid. If the diagram shows the conclusion to be uncertain or false, then the argument is invalid.

5. If all the statements in the argument are universal statements, then the argument can be analyzed in terms of transitive reasoning or false chains (see Unit 2 Module 1), and so diagramming is unnecessary.

6. If both premises are universal statements but the conclusion is an existential statement, then the argument is invalid. No diagram is necessary. You cannot deduce "some" from "all" or "none."
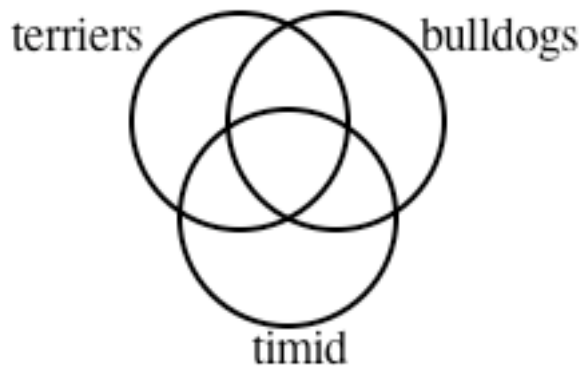
EXAMPLE A
Use diagramming to test the validity of this argument:
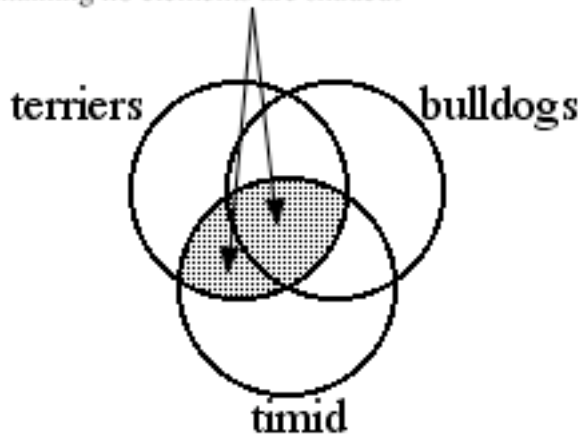No terriers are timid. Some bulldogs are terriers. Therefore, some bulldogs are not timid.

SOLUTION
We will mark this three-circle Venn diagram, which shows the sets "terriers," "bulldogs" and "timid (things):"

First, diagram the negative universal premise "No terriers are timid." According to this premise, the overlap of those two sets contains no elements, so that part of the diagram is shaded, or "blotted out."

Because "No terriers are timid," the intersection
of those two circles contains no elements. Regions
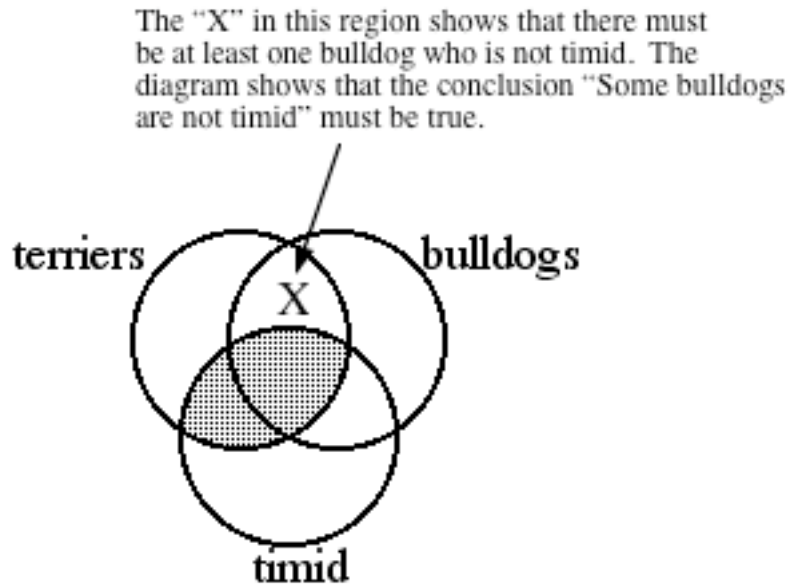containing no elements are shaded.

Next, diagram the existential premise "Some bulldogs are terriers" by placing an
"X" in the appropriate location in the unshaded portion of the diagram.

Because "Some bulldogs are terriers,:" there must
be at least one element in the unshaded
region where those two circles intersect..
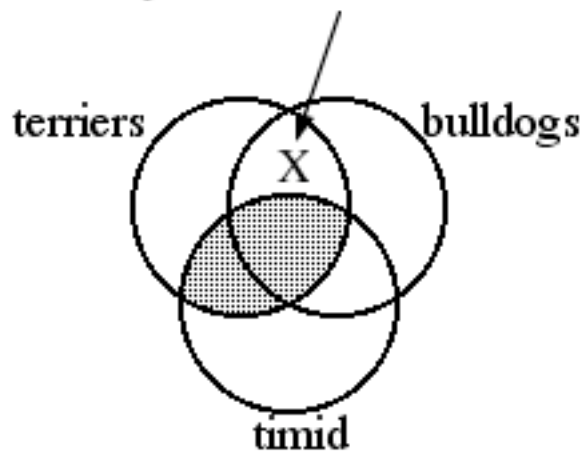
Now that both premises have been diagrammed, check to see if the marked diagram
shows the conclusion to be true.

The "X" in this region shows that there must
be at least one bulldog who is not timid. The
diagram shows that the conclusion "Some bulldogs
are not timid" must be true.

Because the marked diagram shows that the conclusion is true, the argument is
valid.
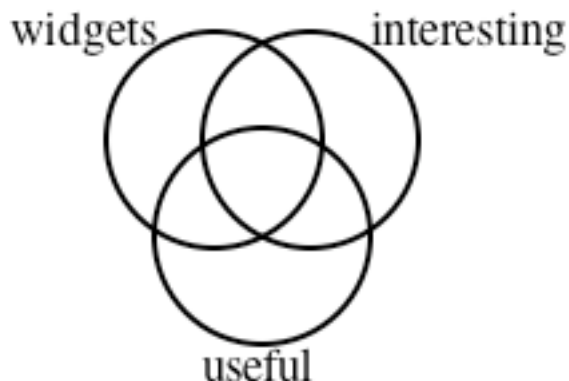
EXAMPLE B
Use diagramming to test the validity of this argument.

Some useful things are interesting. All widgets are interesting. Therefore, some
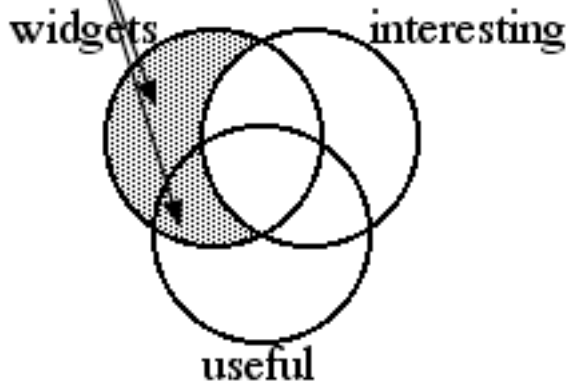widgets are useful.

SOLUTION
We can use this three-circle Venn diagram, shows the sets of widgets, interesting
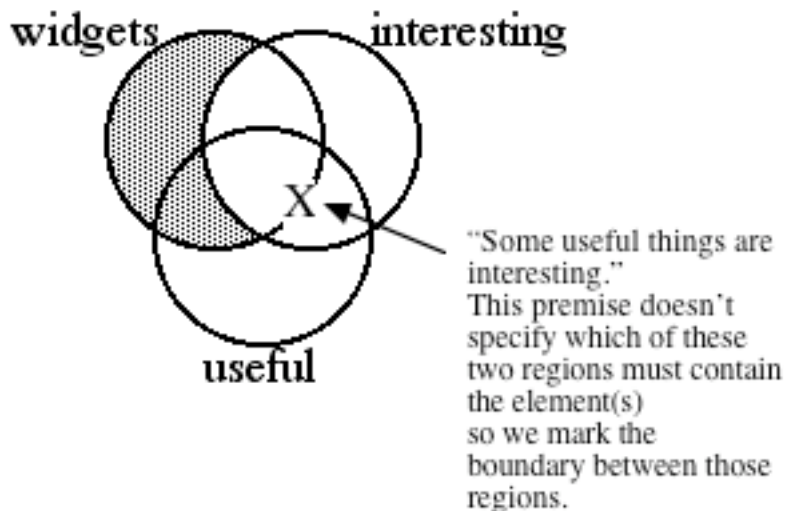(things) and useful (things):

Start by diagramming the universal premise, "All widgets are interesting."

Since "All widgets are interesting,"
this region, where "widgets" are not
"interesting," must contain no elements,
so we shade it.

Next, diagram the existential premise, "Some useful things are interesting." This means that there must be at least one element in the overlap of those two circles. However, that overlap entails two regions, and it is uncertain as which of those two regions contains the element(s), so we place an "X" on their border.

"Some useful things are
interesting."
This premise doesn't
specify which of these
two regions must contain
the element(s)
so we mark the
boundary between those
regions.

Now that we have diagrammed both premises, we check to see if the marked diagram shows the conclusion, "Some widgets are useful," to be true.

In this diagram, the conclusion "Some widgets are useful" is uncertain, so the argument is INVALID.
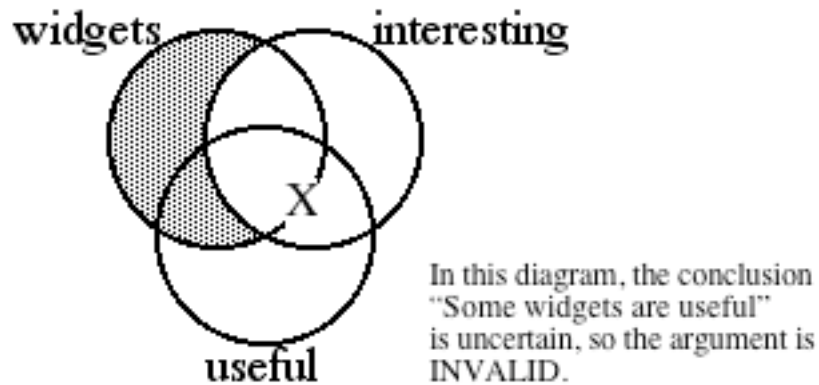
The argument is **invalid**, because the diagram shows that, based on those premises, the conclusion is not certain. That is, the "X" is not in the part of the diagram where "widgets" and "useful" intersect.

EXAMPLE 2.3.1
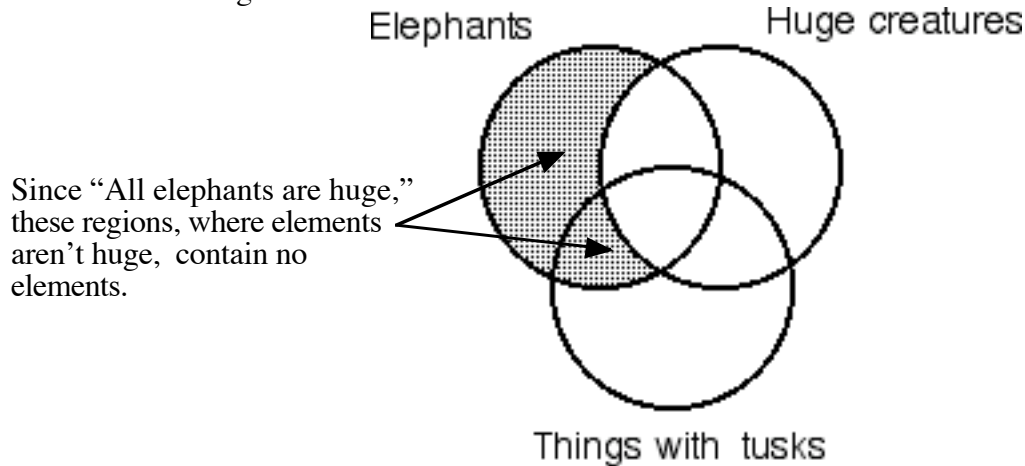*Test the validity of this argument:*
All elephants are huge creatures.
Some huge creatures have tusks.
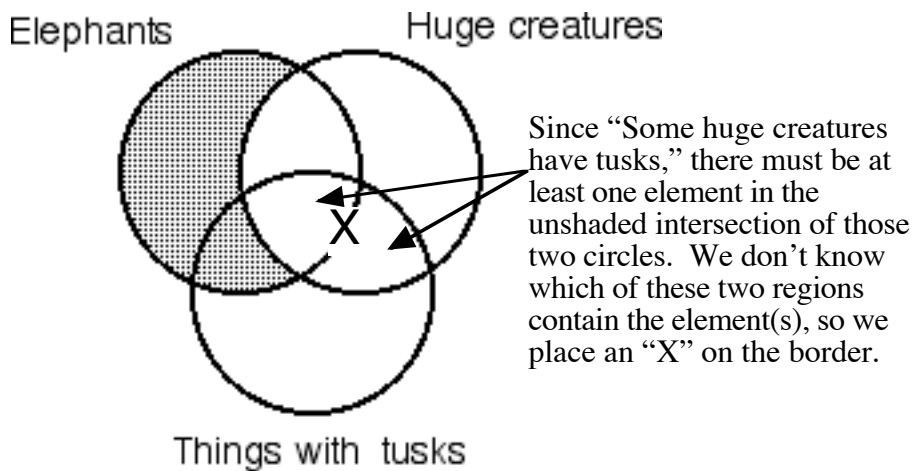Therefore, some elephants have tusks.
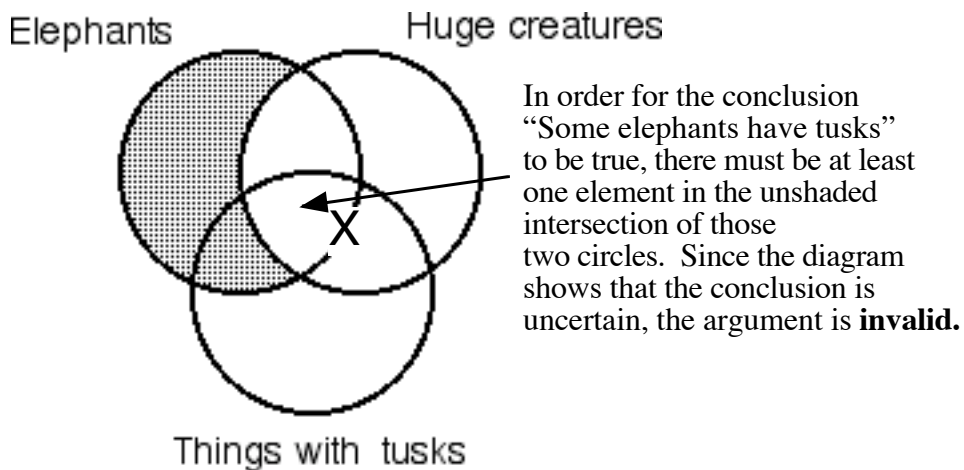
EXAMPLE 2.3.1 SOLUTION
Use shading to diagram the universal premise "All elephants are huge creatures." Shading indicates that a region has no elements.

Elephants   Huge creatures

Since "All elephants are huge," these regions, where elements aren't huge, contain no elements.

Things with tusks

Next, use an "X" to diagram the existential premise "Some huge creatures have tusks."

Elephants   Huge creatures

Since "Some huge creatures have tusks," there must be at least one element in the unshaded intersection of those two circles. We don't know which of these two regions contain the element(s), so we place an "X" on the border.

Things with tusks

Now that both premises have been diagrammed, if the marked diagram shows that the conclusion is true, then the argument is valid. If the marked diagram shows that the conclusion is false or uncertain, then the argument is invalid.

Elephants   Huge creatures

In order for the conclusion "Some elephants have tusks" to be true, there must be at least one element in the unshaded intersection of those two circles. Since the diagram shows that the conclusion is uncertain, the argument is **invalid.**
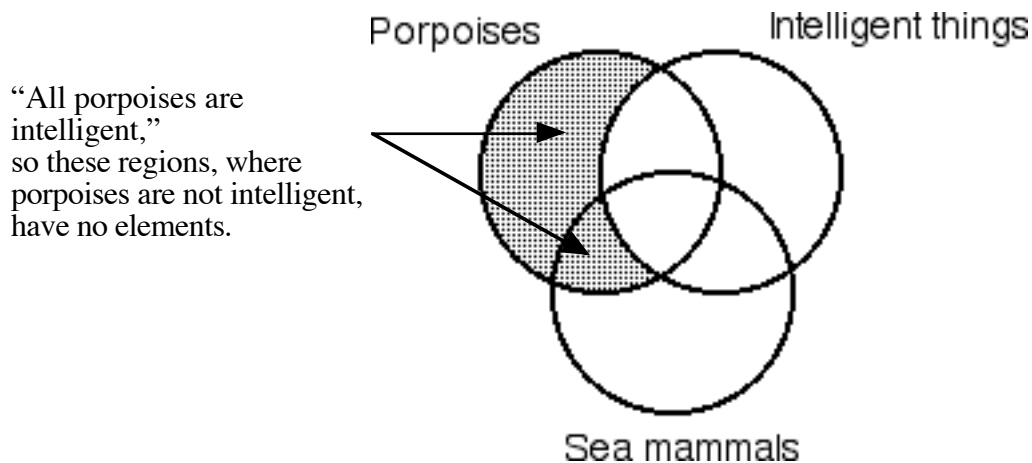
Things with tusks

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EXAMPLE 2.3.2
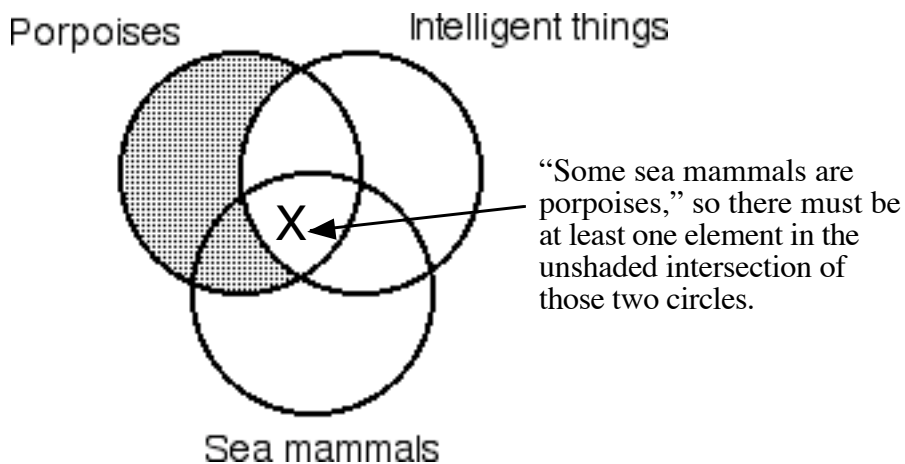Test the validity of the argument:
All porpoises are intelligent. Some sea mammals are porpoises. Therefore, some sea mammals are intelligent.

EXAMPLE 2.3.2 SOLUTION

First use shading to diagram the universal premise.

"All porpoises are intelligent,"
so these regions, where porpoises are not intelligent, have no elements.

Next use an "X" to diagram the existential premise.

"Some sea mammals are porpoises," so there must be at least one element in the unshaded intersection of those two circles.

If the marked diagram shows that the conclusion is true, then the argument is valid. If the marked diagram shows that the conclusion is uncertain or false, then the argument is invalid.

Because there is an "X" in the unmarked intersection of "Sea mammals" and "Intelligent things," the diagram shows that the conclusion, "Some sea mammals are intelligent," is true. The argument is **valid.**
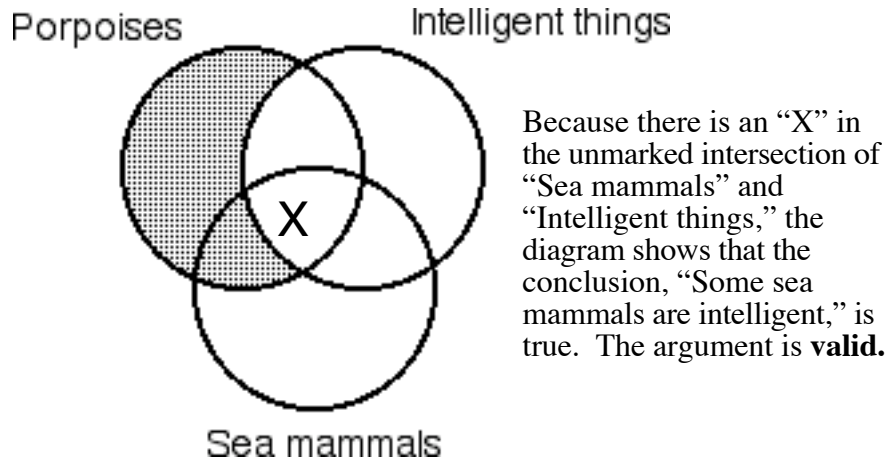
EXAMPLE 2.3.3
Test the validity of the argument:
All cows like to chew. Some dairy animals don't like to chew.
Therefore, some dairy animals aren't cows.

EXAMPLE 2.3.4
Test the validity of the argument:
Some lawyers are judges. Some judges are politicians. Therefore, some lawyers are politicians.

WORLD WIDE WEB NOTE
For practice on arguments involving categorical syllogisms and diagramming, visit the companion website and try The CATEGORIZER.

EXAMPLE 2.3.6
Test the validity of each argument.
A. Some fish are tasty. All fish can swim. Therefore, some tasty things can swim.
B. Some doctors are dentists. Some dentists are surgeons. Therefore, some doctors are surgeons.
C. All hogs are smelly. Some swine aren't hogs. Therefore, some swine aren't smelly.
D. All burglars are criminals. Some thieves are criminals. Therefore, some burglars are thieves.
E. Some food preparers aren't cooks. All chefs are cooks. Some food preparers aren't chefs.
F. No thieves are saintly. Some congressmen are thieves. Therefore, some congressmen aren't saintly.

EXAMPLE 2.3.7
Consider these premises:
All poodles are dogs. All dogs bark.

We should easily recognize that a valid conclusion is "All poodles bark."

Question: since "**All** poodles bark" is a valid conclusion, wouldn't "**Some** poodles bark" also be a valid conclusion? After all, "some" sounds like a softer condition than "all," so common sense suggests that "some A are B" should be a valid conclusion whenever "all A are B" is a valid conclusion.

However, the answer to the question above is "no." In fact, "all" does not imply "some," due to this peculiarity of the word "all:" A statement like "All poodles bite" is true even if there were no poodles. In other words, if all of the poodles went extinct, the statement "All poodles bite" would still be (vacuously) true, but the statement "Some poodles bite" would be false (because "some poodles bites" means that there must be at least one poodle).

This example gives rise to the following observation, which holds for all arguments.
If every premise in an argument has the form "All are...," a valid conclusion will not have the form "Some are..." (unless a premise specifies that all of the sets in the argument are non-empty).

PRACTICE EXERCISES
1 - 18: Test the validity of each argument.

1. All horses have hooves. Some horses eat oats. Therefore, some oat-eaters have hooves.

2. Some mammals are bats. All bats can fly. Thus, some mammals can fly.

3. No dogs can talk. Some searchers are dogs. Thus, some searchers can't talk.

4. Some squirrels fly. Some squirrels gather acorns. Therefore, some acorn-gatherers fly.

5. Some scavengers eat road-kill. All crows eat road-kill. Therefore, all crows are scavengers.

6. All successful politicians are persuasive. Some lawyers are persuasive. Thus, some lawyers are successful politicians.

7. Some skaters drink Surge. Some skaters drink Citra. Therefore, some Surge drinkers are Citra drinkers.

8. All tattooers are body-artists. Some tattooers drive Harleys. Therefore, some body-artists drive Harleys.

9. Some multiple-choice questions are tricky. No easy questions are tricky. Therefore, some multiple-choice questions are not easy.

10. Some snails live under rocks. All snails are slimy. Therefore, some things that live under rocks are slimy.

11. All primates are curious. Some primates are carnivores. Thus, some carnivores are curious.

12. All astronauts are bold. Some pilots are not bold. Therefore, some astronauts are not pilots.

13. Some ants are aggressive. All ants are insects. Therefore, some insects are aggressive.

14. All plumbers use monkey wrenches. Some mechanics don't use monkey wrenches. Hence, some mechanics aren't plumbers.

15. All spies are secretive. Some agents aren't spies. Therefore, some agents aren't secretive.

16. Some poodles yap too much. Some dogs are poodles. Therefore, some dogs yap too much.

17. Some poodles are dogs. All poodles yap too much. Thus, some dogs yap too much.

18. All senators are politicians. Some corrupt people aren't politicians. Therefore, some corrupt people aren't senators.

ANSWERS TO LINKED EXAMPLES
EXAMPLE 2.3.3 Valid
EXAMPLE 2.3.4 Invalid
EXAMPLE 2.3.6   A. Valid   B. Invalid   C. Invalid
                     D. Invalid   E. Valid   F. Valid

ANSWERS TO PRACTICE EXERCISES
1. Valid   **2.** Valid   **3.** Valid   **4.** Invalid   **5.** Invalid   **6.** Invalid
7. Invalid   **8.** Valid   **9.** Valid   **10.** Valid   **11.** Valid   **12.** Invalid
13. Valid   **14.** Valid   **15.** Invalid   **16.** Invalid   **17.** Valid   **18.** Valid