UNIT 2 MODULE 3
EULER DIAGRAMS AND ARGUMENTS INVOLVING THE WORD "SOME"

EXAMPLE 2.3.1
Test the validity of this argument:
All elephants are huge creatures.
Some huge creatures have tusks.
Therefore, some elephants have tusks.

Before we solve this problem, we will point out that this argument is different from any of the arguments that we've encountered so far, in that it involves the word **"some."**
In our previous work we have relied on the following facts that give the relationships between certain quantified statements and logical connectives.

"All A are B" is equivalent to "If A, then B."
"No A are B" is equivalent to "If A, then not B."
However, the quantifier "some" is not covered by the two rules shown above.

FACT: The statement "Some A are B" is not equivalent to any useful statement involving logical connectives.

Since our construction of truth tables depends upon the properties of logical connectives, the fact above leads to this profound fact about arguments:
FACT: The truth table method is of no practical use when an argument contains at least one statement of the form "Some A are B."

This means that we cannot use a truth table to analyze the argument in EXAMPLE 2.3.1 above. We introduce a different technique for use in cases like this:

ANALYZING ARGUMENTS INVOLVING THE WORD "SOME"

When an argument involves the word "some," we use Venn diagrams to try to show that the argument is INVALID.

An argument is INVALID if we are able to draw a Venn diagram that agrees with every PREMISE but denies the CONCLUSION.

Venn diagrams that are used to analyze arguments are usually called ***Euler diagrams***, in honor of the mathematician Leonhard Euler.

A Venn diagram (Euler diagram) that agrees with every premise but denies the conclusion is called a *counterexample* to the argument.

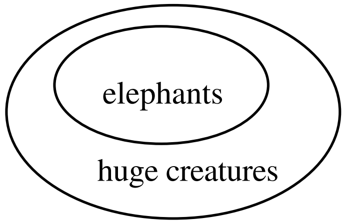
EXAMPLE 2.3.1 SOLUTION

In order to test the validity of the argument
"All elephants are huge creatures.
Some huge creatures have tusks.
Therefore, some elephants have tusks."
we will try to draw a counterexample. That is, we will try to draw an Venn diagram having circles representing the sets "elephants," "huge creatures," and "creatures having tusks."

The argument will be invalid if and only if it is possible to draw the diagram in such a way that it shows that "all elephants are huge creatures" and also shows that "some huge creatures have tusks" but **doesn't** show that "some elephants have tusks."

It is also important to remember this basic fact about all arguments:
Whether or not an argument is valid has nothing to do with whether or not the conclusion sounds like a reasonable statement.

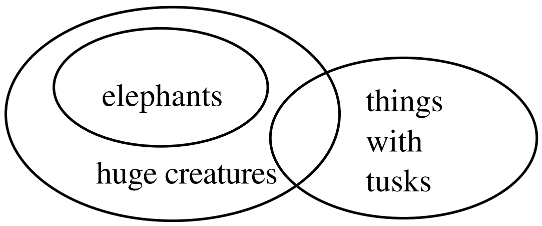
We begin with a Venn diagram that agrees with the first premise, "All elephants are huge creatures. This diagram must show the set "elephants" contained within the set "huge creatures."



All elephants are huge creatures.

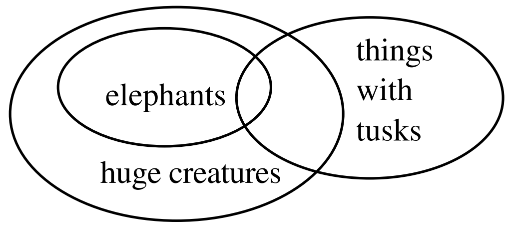
Next, we expand the diagram to include the second premise, "Some huge creatures have tusks." Thus, we must draw the set of "things with tusks" in such a way that it overlaps the set of huge creatures. In fact there are several ways to do this, including:

I



All elephants are huge creatures.
Some huge creatures have tusks.

II



All elephants are huge creatures.
Some huge creatures have tusks.

Notice that although both Figures I and II above agree with both premises, Figure I also **denies the conclusion** (that it, it doesn't show that "Some elephants have tusks"). **Figure I above is a counterexample to the argument; it shows us that the argument is INVALID.**

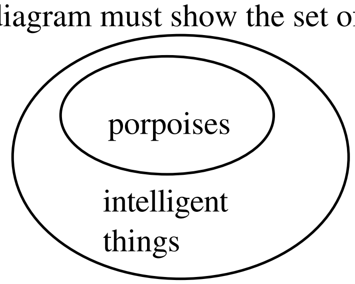
In fact, the invalid argument in this example is closely related to Fallacy of the Converse.

EXAMPLE 2.3.2
Test the validity of the argument:
All porpoises are intelligent. Some sea mammals are porpoises. Therefore, some sea mammals are intelligent.

EXAMPLE 2.3.2 SOLUTION

We will try to draw an Euler diagram that is a counterexample to the argument; that is, we will try to show that the argument is invalid by drawing a diagram that shows "all porpoises are intelligent" and also shows "some sea mammals are porpoises" but **doesn't** show the conclusion "some sea mammals are intelligent."

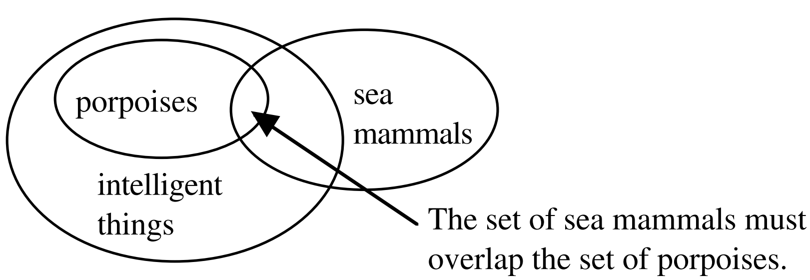
We start with a diagram for the first premise, "All porpoises are intelligent." This diagram must show the set of porpoises contained within the set of intelligent things.



All porpoises are intelligent.

Next, we expand the diagram to include the set of sea mammals in such a way it agrees with the statement "Some sea mammals are porpoises." At the same time, however, we want to try to deny the conclusion "Some sea mammals are intelligent."

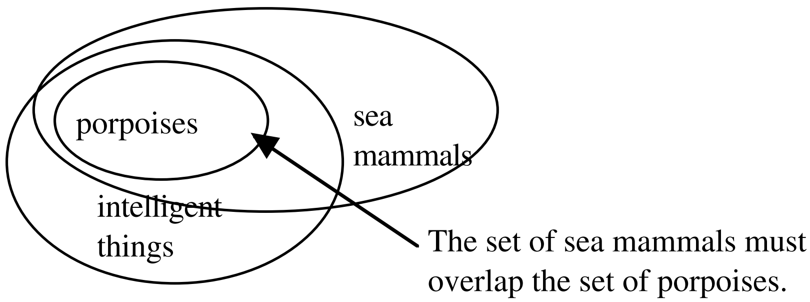
I



All porpoises are intelligent.
Some sea mammals are porpoises.

Here is another diagram that agrees with both premises.

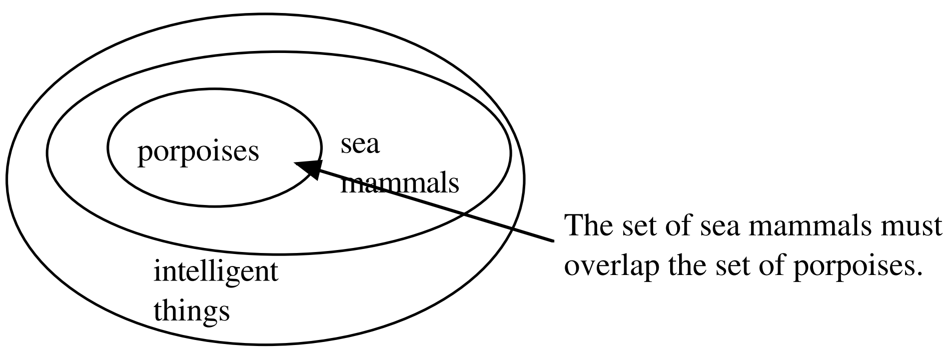
II



All porpoises are intelligent.
Some sea mammals are porpoises.

Here is yet another diagram that agrees with both premises.

III



All porpoises are intelligent.
Some sea mammals are porpoises.

In this case, *we will find that no matter how we try, any diagram that agrees with the two premises with* ***automatically*** *agree with the conclusion.*

We have determined that the argument is VALID.
It is important to understand that the argument is valid *not because of the particular diagrams shown above*, but because in drawing that diagram we realized that *a counterexample is not possible.*
In fact, the argument in this example is closely related to Direct Reasoning.

Fact: In trying to use Euler diagrams to analyze a valid argument, we must come to the realization that it is not possible to draw a counterexample.

EXAMPLE 2.3.3
Test the validity of the argument:
All cows like to chew. Some dairy animals don't like to chew.
Therefore, some dairy animals aren't cows.

EXAMPLE 2.3.4
Test the validity of the argument:
Some lawyers are judges. Some judges are politicians. Therefore, some lawyers are politicians.

WORLD WIDE WEB NOTE
For practice on arguments involving Euler diagrams and the word “some,” visit the companion website and try “SOME” ARGUMENTS.

EXAMPLE 2.3.6
Test the validity of each argument.
A. Some fish are tasty. All fish can swim. Therefore, some tasty things can swim.
B. Some doctors are dentists. Some dentists are surgeons. Therefore, some doctors are surgeons.
C. All hogs are smelly. Some swine aren't hogs. Therefore, some swine aren't smelly.
D. All burglars are criminals. Some thieves are criminals. Therefore, some burglars are thieves.
E. Some food preparers aren't cooks. All chefs are cooks. Some food preparers aren't chefs.
F. No thieves are saintly. Some congressmen are thieves. Therefore, some congressmen aren’t saintly.

EXAMPLE 2.3.7
Consider these premises:
All poodles are dogs. All dogs bark.

We should easily recognize that a valid conclusion is "All poodles bark."

Question: since "**All** poodles bark" is a valid conclusion, wouldn't "**Some** poodles bark" also be a valid conclusion? After all, "some" sounds like a softer condition than "all," so common sense suggests that "some A are B" should be a valid conclusion whenever "all A are B" is a valid conclusion.

However, the answer to the question above is "no." In fact, "all" does not imply "some," due to this peculiarity of the word "all:" A statement like "All poodles bite" is true even if there were no poodles. In other words, if all of the poodles went extinct, the statement "All poodles bite" would still be (vacuously) true, but the statement "Some poodles bite" would be false (because "some poodles bites" means that there must be at least one poodle).

This example gives rise to the following observation, which holds for all arguments. **If every premise in an argument has the form "All are...," a valid conclusion will not have the form "Some are..."** (unless a premise specifies that all of the sets in the argument are non-empty).

HOMEWORK EXERCISES

1 - 18: Test the validity of each argument.

1. All horses have hooves. Some horses eat oats. Therefore, some oat-eaters have hooves.

2. Some mammals are bats. All bats can fly. Thus, some mammals can fly.

3. No dogs can talk. Some searchers are dogs. Thus, some searchers can't talk.

4. Some squirrels fly. Some squirrels gather acorns. Therefore, some acorn-gatherers fly.

5. Some scavengers eat road-kill. All crows eat road-kill. Therefore, all crows are scavengers.

6. All successful politicians are persuasive. Some lawyers are persuasive. Thus, some lawyers are successful politicians.

7. Some skaters drink Surge. Some skaters drink Citra. Therefore, some Surge drinkers are Citra drinkers.

8. All tattooers are body-artists. Some tattooers drive Harleys. Therefore, some body-artists drive Harleys.

9. Some multiple-choice questions are tricky. No easy questions are tricky. Therefore, some multiple-choice questions are not easy.

10. Some snails live under rocks. All snails are slimy. Therefore, some things that live under rocks are slimy.

11. All primates are curious. Some primates are carnivores. Thus, some carnivores are curious.

12. All astronauts are bold. Some pilots are not bold. Therefore, some astronauts are not pilots.

13. Some ants are aggressive. All ants are insects. Therefore, some insects are aggressive.

14. All plumbers use monkey wrenches. Some mechanics don't use monkey wrenches. Hence, some mechanics aren't plumbers.

15. All spies are secretive. Some agents aren't spies. Therefore, some agents aren't secretive.

16. Some poodles yap too much. Some dogs are poodles. Therefore, some dogs yap too much.

17. Some poodles are dogs. All poodles yap too much. Thus, some dogs yap too much.

18. All senators are politicians. Some corrupt people aren't politicians. Therefore, some corrupt people aren't senators.

ANSWERS TO LINKED EXAMPLES

EXAMPLE 2.3.3 Valid

EXAMPLE 2.3.4 Invalid

EXAMPLE 2.3.6

A. Valid	B. Invalid	C. Invalid
D. Invalid	E. Valid	F. Valid

ANSWERS TO HOMEWORK EXERCISES

1. Valid	**2.** Valid	**3.** Valid	**4.** Invalid	**5.** Invalid	**6.** Invalid
7. Invalid	**8.** Valid	**9.** Valid	**10.** Valid	**11.** Valid	**12.** Invalid
13. Valid	**14.** Valid	**15.** Invalid	**16.** Invalid	**17.** Valid	**18.** Valid