Part 2 Module 2 Extension: The Biconditional and the Exclusive Or

Two common sources of error in logic involve misusing conditional statements and misuing disjunctions.

A typical misuse of conditional statements is confusing a conditional with its converse or its inverse.

A typical misuse of disjunctions is failure to realize that "or" in logic is inclusive.

Anothrer way of stating it is to say that a typical error in logic is confusing a conditional statement with a ***biconditional*** statement, and a second typical error is confusing a disjunction with an ***exclusive disjunction***.

Biconditional statements

A ***biconditional*** statement is a statement of the form "*p, if and only if q*."

This is denoted $p \leftrightarrow q$, and is sometimes abbreviated "*p iff q*."

Definition*: A biconditional statement is true, only when the two terms have the same value.*

Exclusive disjunctions

An ***exclusive disjunction***, more simply called an ***exlcusive or***, is a statement of the form "*p or q (but not both)*."

This is denoted $p \oplus q$, and is sometimes abbreviated "*p xor q*."

Definition: *An exclusive or statement is true, only when exactly one of the two terms is true.*

This truth table illustrates the definitions of the *biconditional* and *exclusive or* propositions.

p	q	$p \leftrightarrow q$	$p \oplus q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

Exercises

1-3: Use a truth table to prove each of the following:

1. $p \leftrightarrow q \equiv \left(p \rightarrow q\right) \wedge \left(q \rightarrow p\right)$

2. $p \leftrightarrow q \equiv \left(p \rightarrow q\right) \wedge \left(\sim p \rightarrow \sim q\right)$

3. $\sim\left(p \leftrightarrow q\right) \equiv p \oplus q$

4. Let p be the statement "Thomasville is the capitol of Georgia."
Let q be the statement "Sopchoppy is the capitol of Florida."

Determine the truth values of:

a. $p \rightarrow q$

b. $\neg p \rightarrow q$

c. $p \leftrightarrow q$

d. $\sim p \leftrightarrow q$

e. $p \vee q$

f. $\sim p \vee \sim q$

g. $p \oplus q$

h. $\sim p \oplus \sim q$

Answers

1.

p	q	~p	~q	p↔q	p→q	q→p	(p→q)∧(q→p)
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	F	F	T	F	F
F	F	T	T	T	T	T	T

2.

p	q	~p	~q	p↔q	p→q	~p→~q	(p→q)∧(~p→~q)
T	T	F	F	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	F	F	T	F	F
F	F	T	T	T	T	T	T

3.

p	q	~p	~q	p↔q	~ p↔q	p⊕q
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

4.

a. $p \rightarrow q$ **T**

b. $\neg p \rightarrow q$ **F**

c. $p \leftrightarrow q$ **T**

d. $\sim p \leftrightarrow q$ **F**

e. $p \vee q$ **F**

f. $\sim p \vee \sim q$ **T**

g. $p \oplus q$ **F**

h. $\sim p \oplus \sim q$ **F**