PART 3 MODULE 5 extension
MATHEMATICAL EXPECTATION

If the outcomes of a random experiment are all *numbers*, then we can define the *expected value* of the experiment.
The expected value is like the "average outcome."

Suppose that the sample space *S* for a random experiment is the set of ***numbers*** $x_1, x_2, \ldots, x_n$.
If $P(x_1), P(x_2), \ldots, P(x_n)$ are the respective probabilities, then we define the **EXPECTED VALUE** (EV) as the sum of all products formed by multiplying a numerical outcome by its probability.
$EV = x_1 \times P(x_1) + x_2 \times P(x_2) + \ldots + x_n \times P(x_n)$.

EXAMPLE

Suppose a gambling contest works as follows: one die is rolled. If the result of the die roll is "1" or "2" the player wins $1; if the die roll is "3" or "4" the player wins $3; if the die roll is "5" the player wins $10; if the die roll is "6" the player loses $25. Find the expected value.

SOLUTION

We will summarize the contest with the following **Payout Table**

x	*P(x)*
$1	2/6
$3	2/6

$10	1/6
–$25	1/6

To find the **EXPECTED VALUE** we find the sum of all terms formed by multiplying a payout times its probability.

Expected Value = ($1×2/6) + ($3×2/6) + ($10×1/6) + (–$25×1/6)
= ($2+$6+$10–$25)/6
= –$7/6
= –$1.17

This value represents the **long-term average payout.** In other words, if a person played the game many, many times, in the long run the person would average a loss of $1.17 per play. For instance, if a person played the game 1000 times, he or she would expect to lose roughly $1,170.

EXAMPLE

A carnival game works as follows: On a table there are a number of drinking glasses; each glass contains water and a live goldfish. The contestant attempts to toss a dime so that it lands in any one of the glasses. If successful, the contestant wins the goldfish contained therein. In unsuccessful, the contestant loses the dime. The goldfish is worth 5¢. The game operator assumes that a contestant will succeed once in every five tries. From the perspective of the person operating the game, find the expected value of this contest.

ANSWER

If the contestant doesn't win a fish, the game operator experiences a "payout" of 10 cents; the probability that this will occur is 4/5.
If the contestant wins a fish, the game operator only experiences a "payout" of 5 cents (since the operator keeps the contestant's dime but forfeits a 5-cent fish); the probability of this payout is 1/5.

x	$P(x)$
10¢	.8
5¢	.2

Expected value = (10¢×.8) + (5¢×.2)
= 8¢ + 1¢
= 9¢

On average the game operator can expect to gain a 9¢ profit each time the game is played. If the game is played 1,000 times, the operator can expect a proft of $90.

EXAMPLE

A professional bowler is going to purchase a $500,000 insurance policy on his thumb, in case his career should be cut short by a debilitating thumb injury. The Gomermatic Corporation specializes in such policies. Their actuaries estimate that in general, the probability that a bowler will suffer a debilitating thumb injury is .0002. The company will charge $10,000 for the insurance policy. From the perspective of the Gomermatic Corporation, what is the expected value of such a policy?

ANSWER

If the bowler doesn't suffer a debilitating thumb injury, the corporation gains $10,000. The probability of this gain is .9999. If the bowler does suffer such an injury, the company loses $490,000 (they keep the $10,000 that was initially paid). The probability of this loss is .0002. The expected value is:

$$.9998(\$10{,}000) + (.0002)(-\$490{,}000) = \$9{,}900$$

If the company issues a large number of these policies, they can expect to earn an average of $9,900 per policy.