PART 1 MODULE 2
SET OPERATIONS, VENN DIAGRAMS

SET OPERATIONS

Let U = {x|x is an English-language film}
Set A below contains the five best films according to the American Film Institute.
A = {*Citizen Kane, Casablanca, The Godfather, Gone With the Wind, Lawrence of Arabia*}

Set B below contains the five best films according to TV Guide.
B= {*Casablanca, The Godfather Part 2, The Wizard of Oz, Citizen Kane, To Kill A Mockingbird*}

Set C below contains the five most passionate films according to the American Film Institute.
C = {*Gone With the Wind, Casablanca, West Side Story, An Affair To Remember, Roman Holiday*}.

Notice that some films appear on more than one of these lists.

SET INTERSECTION AND SET UNION

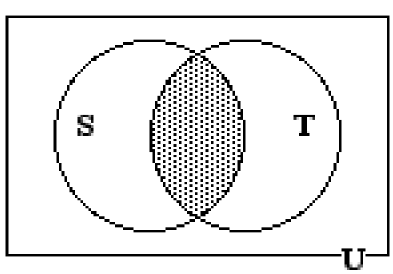
Casablanca and *Citizen Kane* are the films that are simultaneously in sets A and B. We say that {*Casablanca, Citizen Kane*} is the **intersection** of A and B.

This is denoted:
A ∩ B = {*Casablanca, Citizen Kane*}

Likewise,
A ∩ C ={*Gone With the Wind, Casablanca*}   B ∩ C = {*Casablanca*}
In general, *if S and T are sets then* $S \cap T = \{x | x \in S$ **and** $x \in T\}$.

A **Venn diagram** is a drawing in which geometric figures such as circles and rectangles are used to represent sets. One use of Venn diagrams is to illustrate the effects of set operations.

The shaded region of the Venn diagram below corresponds to S ∩ T



$S \cap T$

Now suppose we merge all of the elements of A with all of the elements of B to form a single, larger set:
{*Citizen Kane, Casablanca, The Godfather, Gone With the Wind, Lawrence of Arabia, The Godfather Part 2, The Wizard of Oz, To Kill A Mockingbird*}

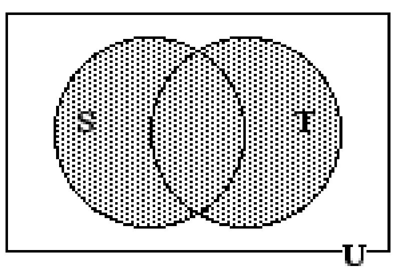
This new set is called the **union** of A with B, and is denoted: $A \cup B$

Likewise,
$A \cup C$ = {*Citizen Kane, Casablanca, The Godfather, Gone With the Wind, Lawrence of Arabia*, *West Side Story, An Affair To Remember, Roman Holiday*}
and
$B \cup C$ = {*Casablanca, The Godfather Part 2, The Wizard of Oz, Citizen Kane, To Kill A Mockingbird*, *Gone With the Wind, Casablanca, West Side Story, An Affair To Remember, Roman Holiday*}.

Note that in listing the elements of the sets above, the order in which the films were listed did not matter, and it doesn't make sense to list the same film more than once within the same set.

In general, *for any sets S and T,* $S \cup T = \{x | x \in S$ ***or*** $x \in T\}$.

The shaded part of the Venn diagram below illustrates $S \cup T$



$S \cup T$

SET INTERSECTION, SET UNION, SET COMPLEMENT: SUMMARY

The **intersection** of two sets denotes the elements that the sets have in common, or the "overlap" of the two sets.

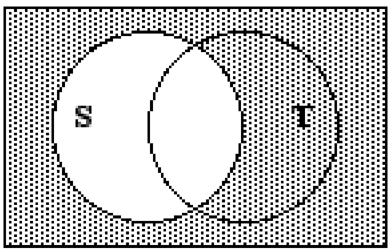
$S \cap T = \{x | x \in S \text{ and } x \in T\}$.

The **union** of two sets merges the two sets into one "larger" set.

$S \cup T = \{x | x \in S \text{ or } x \in T\}$.

The **complement** of a set consists of all elements in the universal set that are not in the given set.

$S' = \{x | x \notin S\}$.



S′

EXAMPLE 1.2.1

For problems 1 - 10 refer to these sets:

U = {a, b, c, d, e, f}   A = {a, c, e, f}   B = {c, d, e}   C = {e, f}

Find each of the following:

1. A'

2. B'

3. C'

4. $B \cup C$

5. $A \cap C$

6. $B \cap C$

7. $(A \cup B)'$

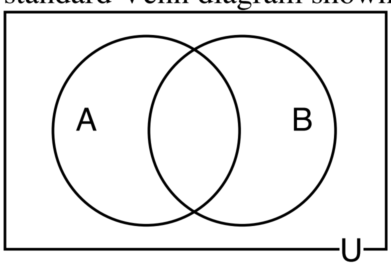
8. $A' \cup B'$

9. $B' \cap C$

10. $A \cup (B' \cap C)$

World Wide Web note:
For more practice exercises involving set operations, visit the companion web site and try THE BIG OPERATOR.

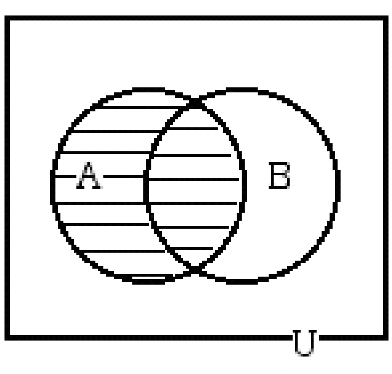
Note: In problems 11 - 16 that follow, the sets A, B, C and U are not the same sets that were used problems 1 - 10. These problems are to be solved in general, not by referring to specific sets whose elements are known. We will produce a shaded region on the standard Venn diagram shown below.



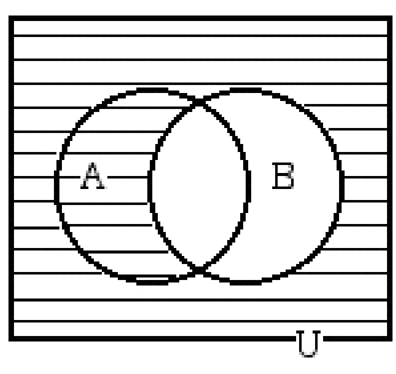
11. On a Venn diagram, shade the region(s) corresponding to $A \cap B$.
12. On a Venn diagram, shade the region(s) corresponding to $A \cup B$.
13. On a Venn diagram, shade the region(s) corresponding to $A \cup B'$.
14. On a Venn diagram, shade the region(s) corresponding to $A \cap B'$.
15. On a Venn diagram, shade the region(s) corresponding to $(A \cup B)'$.
16. On a Venn diagram, shade the region(s) corresponding to $A' \cap B'$.

Solution to Example 1.2.1 #13
To shade the set we need to compare the Venn diagram for A with the Venn diagram for B′, and bear in mind the meaning of **union**.

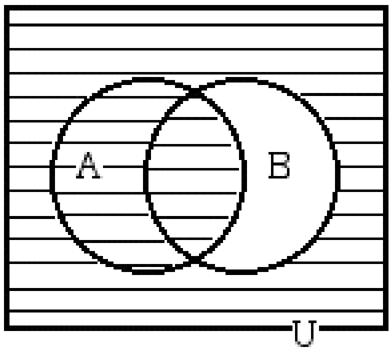
Here is the Venn diagram for set A:



Here is the Venn diagram for set B′:



We combine these two Venn diagrams using set union. This means that any region that is shaded in either of the diagrams above will be shaded in $A \cup B'$.



$A \cup B'$

EXAMPLE 1.2.2
Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
S = {2, 5, 7, 9}
V = {3, 4, 5, 6, 7}
T = {1, 3, 4, 5, 8, 9}
Find $(S' \cap T)'$

The results of exercises 15 and 16 in Example 1.2.2 above suggest the following facts:

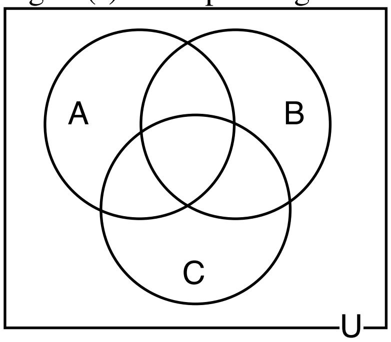
DeMORGAN'S LAWS FOR SET MATHEMATICS

For any sets S, T:
$(S \cup T)' = S' \cap T'$ and $(S \cap T)' = S' \cup T'$

In words:
"The complement of a union is the intersection of the complements."
"The complement of an intersection is the union of the complements."

EXAMPLE 1.2.3

1-6: On a standard three-circle Venn diagram like the one shown below, shade the region(s) corresponding to the given set expression.



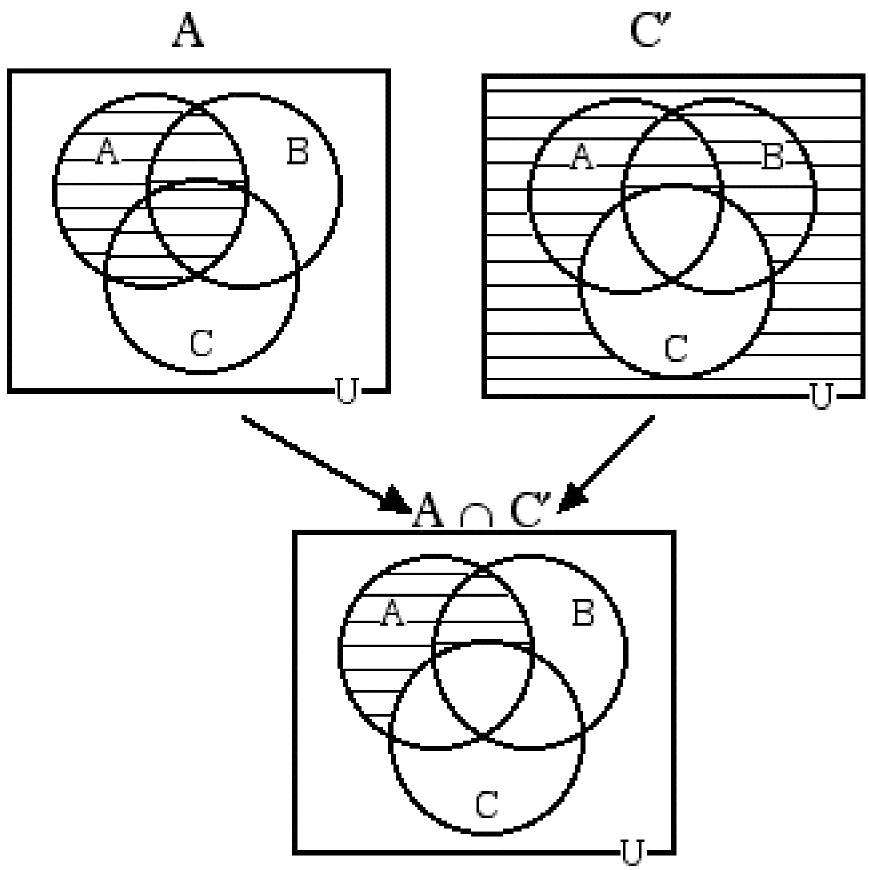
1. $(A' \cap B) \cap C'$
2. $A \cap (B \cap C')$
3. $(A \cup B) \cap C$
4. $(A \cap C') \cup B'$
5. $(A' \cup B)' \cap C$
6. $A' \cup (B' \cap C)$

SOLUTION FOR EXAMPLE 1.2.3 #4

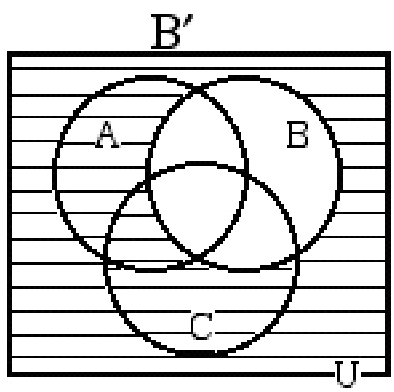
The key to solving a problem like this is to employ a logical process in which, at any step, we never do more than compare to simple objects using one simple rule.

In order to make a Venn diagram for $(A \cap C') \cup B'$, we need to compare the Venn diagram for $A \cap C'$ with the Venn diagram for B' using the simple rule for union. However, in order to do that, we must first make a Venn diagram for $A \cap C'$. We do so by comparing the Venn diagram for A with the Venn diagram for C', using the simple rule for intersection.

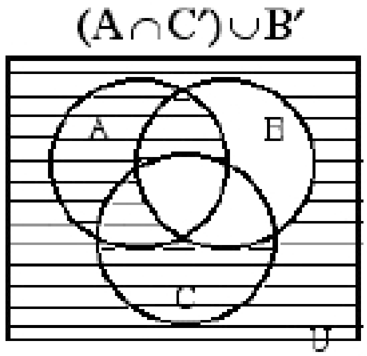
Recall that the intersection of two objects contains the elements that the two objects have in common. This means that in the figure for $A \cap C'$ we will shade any regions that are simultaneously shaded in the figure for A and the figure for C'. Also note that the figure for A will shade everything inside of circle A, while the figure for C' will shade everything outside of circle C.



Now that we know what A ∩ C′ looks like, we compare it with B′.



We compare this figure for B′ with the figure for A ∩ C′ above, using union. Recall that the union of two objects contains all of the elements from both objects. This means that the shaded region for (A ∩ C′) ∪ B′ will contain all shaded regions from A ∩ C′ along with all shaded regions from B′.



WORLD WIDE WEB NOTE
For unlimited practice involving three-circle Venn diagram shading problems, visit the companion web site and try MY COUSIN VENNY.

PRACTICE EXERCISES

1 – 15: U = {b, c, d, e, f, g, h, i, j, k} S = {b, c, d, h, i, k}
T = {b, d, e, f, h} V = {b, d, e, f, g, i}

Find: **1.** S′ **2.** T′ **3.** V′ **4.** S∩T **5.** S∪T **6.** S∩V **7.** S∪V
8. T∩V **9.** T∪V **10.** S′∪V **11.** S′∩V **12.** S′∪T **13.** S′∩T
14. V′∪T **15.** V′∩T

16. Let U = { b, c, d, e, f, g, h, i, j } V = {e} W = {c, f, g, j} Find (V′ ∪ W′)′

17. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 } T = {2, 3, 9} V = {8, 9, 10}
Find (T′ ∪ V)′

18. Let U = { 1, 2, 3, 4, 5, 6, 7, 8 } T = {2, 5} V = {1, 2, 3, 7, 8}
Find (V∩T′)′

19. U = {1, 2, 3, 4, 5, 6, 7} S = {2, 4, 5} T = {3, 5, 7}
V = {2, 3, 4, 5, 7} W = {1, 2, 3, 4, 6} Find (S∩V)′ ∪ (W′∪ T)

20. U = {a, b, c, d, e, f, g} S = {a, b, c, d, e, g}
T = {a, b, f, g} V = {d} W = {a, c, d, e, g} Find [(S∪W) ∪ T′]′

21. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 } S = {1, 2, 5, 6, 7} T = {5, 6}
V = {1, 2, 3, 5, 6, 9} Find S′∪(T∩V)

22 – 33: On a standard 3-circle Venn diagram, shade the region(s) corresponding to the set:

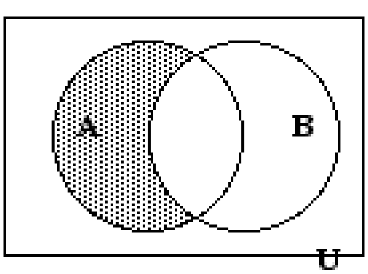
22. (B ∪ A) ∩ C′ **23.** (C ∩ B) ∪ A **24.** C ∪ (A ∩ B′)
25. (A′ ∩ B) ∪ (A ∩ C) **26.** (A′ ∪ B) ∪ C′ **27.** (A ∪ B)′ ∩ C
28. (A ∩ B′) ∪ (B ∩ C′) **29.** (C ∪ A′) ∩ B′ **30.** (B ∩ C′) ∩ A′
31. (B ∩ C′) ∪ A **32.** (A ∪ C) ∩ (A ∪ B) **33.** (A ∩ C)′ ∪ B

ANSWERS FOR LINKED EXAMPLES

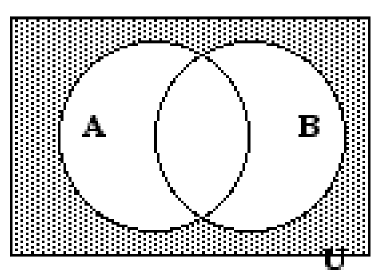
EXAMPLE 1.2.1

1. {b, d} 2. {a, b, f} 3. {a, b, c, d} 4. {c, d, e, f} 5. {e, f}
6. {e} 7. {b} 8. {a, b, d, f} 9. {f} 10. {a, c, e, f}

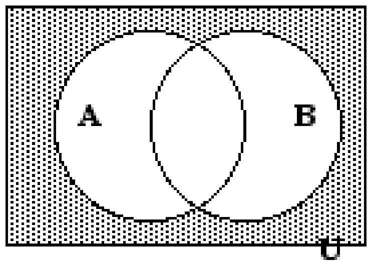
14. A∩B′



15. (A∪B)′



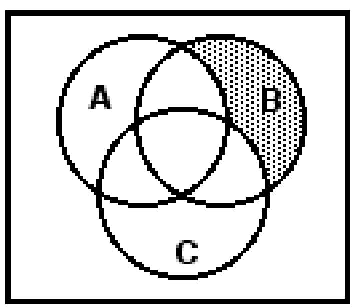
16. $A' \cap B'$



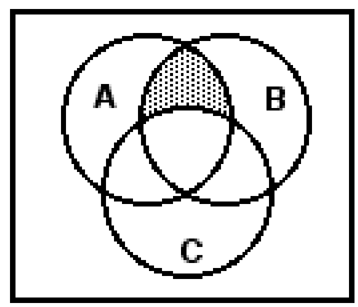
EXAMPLE 1.2.2 {2, 5, 6, 7, 9, 10}

EXAMPLE 1.2.3

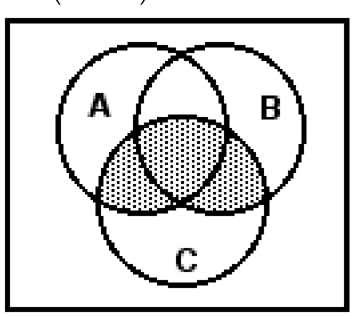
1. $(A' \cap B) \cap C'$



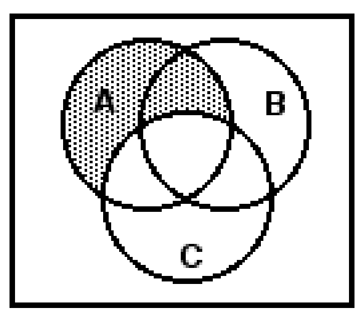
2. $A \cap (B \cap C')$



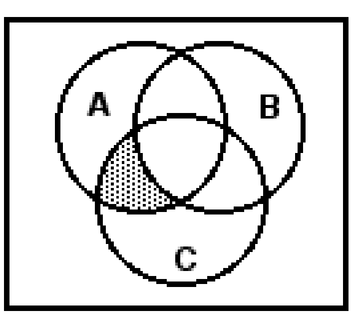
3. $(A \cup B) \cap C$



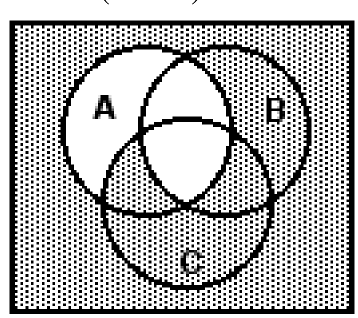
4. $(A \cap C') \cup B'$



5. $(A' \cup B)' \cap C$



6. $A' \cup (B' \cap C)$

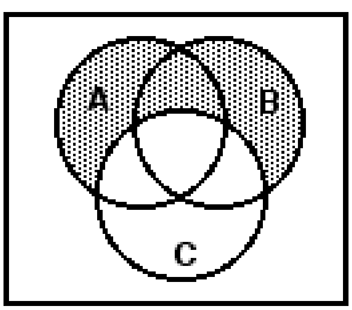


ANSWERS TO PRACTICE EXERCISES

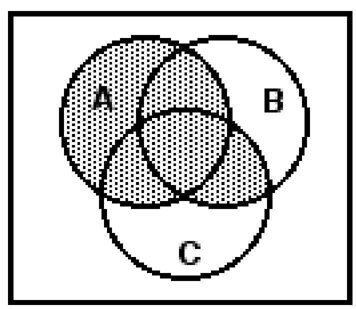
1 – 10: U = {b, c, d, e, f, g, h, i, j, k} S = {b, c, d, h, i, k}
T = {b, d, e, f, h} V = {b, d, e, f, g, i}

1. S′ ={e, f, g, j} **2.** T′={c, g, i, j, k} **3.** V′={c, h, j, k}
4. S∩T = {b, d, h} **5.** S∪T= {b, c, d, e, f, h, i, k} **6.** S∩V= {b, d, i}
7. S∪V= {b, c, d, e, f, g, h, i, k} **8.** T∩V= {b, d, e, f} **9.** T∪V= {b, d, e, f, g, h, i}
10. S′∪V= {b, d, e, f, g, i, j} **11.** S′∩V= {e, f, g} **12.** S′∪T = {b, d, e, f, g, h, j}
13. S′∩T = {e, f} **14.** V′∪T= {b, c, d, e, f, h, j, k} **15.** V′∩T= {h}
16. Let U = { b, c, d, e, f, g, h, i, j } V = {e} W = {c, f, g, j}
(V′ ∪ W′)′ = { }
17. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 } T = {2, 3, 9} V = {8, 9, 10}
(T′ ∪ V)′ = {2, 3}
18. Let U = { 1, 2, 3, 4, 5, 6, 7, 8 } T = {2, 5} V = {1, 2, 3, 7, 8}
(V∩T′)′ = {2, 4, 5, 6}
19. U = {1, 2, 3, 4, 5, 6, 7} S = {2, 4, 5} T = {3, 5, 7}
V = {2, 3, 4, 5, 7} W = {1, 2, 3, 4, 6} (S∩V)′ ∪ (W′∪ T) = {1, 3, 5, 6, 7}
20. U = {a, b, c, d, e, f, g} S = {a, b, c, d, e, g}
T = {a, b, f, g} V = {d} W = {a, c, d, e, g} [(S∪W) ∪ T′]′ = {f}
21. Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 } S = {1, 2, 5, 6, 7} T = {5, 6}
V = {1, 2, 3, 5, 6, 9} S′∪(T∩V) = {3, 4, 5, 6, 8, 9}

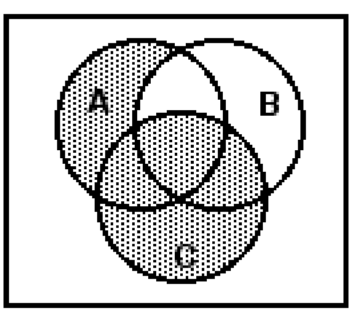
22. (B ∪ A) ∩ C′



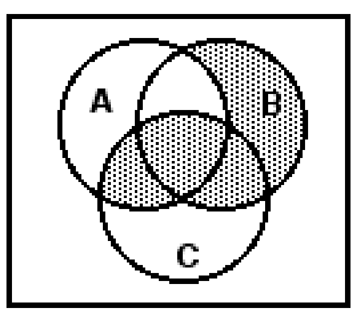
23. (C ∩ B) ∪ A



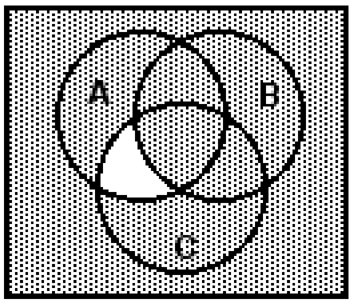
24. C ∪ (A ∩ B′)



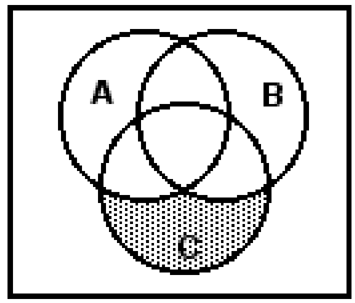
25. (A′ ∩ B) ∪ (A ∩ C)



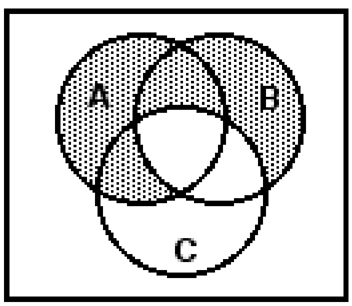
26. $(A' \cup B) \cup C'$



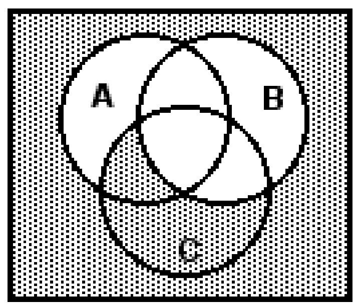
27. $(A \cup B)' \cap C$



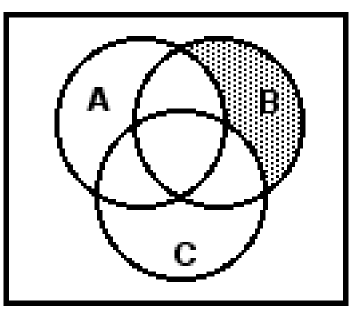
28. $(A \cap B') \cup (B \cap C')$



29. $(C \cup A') \cap B'$



30. $(B \cap C') \cap A'$



31. $(B \cap C') \cup A$