

PART 2 MODULE 1

LOGIC: STATEMENTS, NEGATIONS, QUANTIFIERS, TRUTH TABLES

STATEMENTS

A statement is a declarative sentence having truth value.

Examples of statements:

Today is Saturday.

Today I have math class.

$1 + 1 = 2$

$3 < 1$

~~What's your sign?~~

Some cats have fleas.

All lawyers are dishonest.

Today I have math class and today is Saturday.

$1 + 1 = 2$ or $3 < 1$

For each of the sentences listed above (except the one that is stricken out) you should be able to determine its **truth value** (that is, you should be able to decide whether the statement is TRUE or FALSE).

Questions and commands are not statements.

SYMBOLS FOR STATEMENTS

It is conventional to use lower case letters such as p, q, r, s to represent logic statements.

Referring to the statements listed above, let

p: Today is Saturday.

q: Today I have math class.

r: $1 + 1 = 2$

s: $3 < 1$

u: Some cats have fleas.

v: All lawyers are dishonest.

Note: In our discussion of logic, when we encounter a subjective or value-laden term (an *opinion*) such as "dishonest," we will assume for the sake of the discussion that that term has been precisely defined.

QUANTIFIED STATEMENTS

*The words "all" "some" and "none" are examples of **quantifiers**.*

*A statement containing one or more of these words is a **quantified** statement.*

Note: the word "some" means "at least one."

EXAMPLE 2.1.1

According to your everyday experience, decide whether each statement is true or false:

1. All dogs are poodles.
2. Some books have hard covers.
3. No U.S. presidents were residents of Georgia.
4. Some cats are mammals.
5. Some cats aren't mammals.

EXAMPLE 2.1.1 SOLUTIONS

1. False (because we know that there is at least one dog that is not a poodle).
2. True (because we know that there is at least one book that doesn't have a hard cover).
3. False (because we know that there was at least one president who was from Georgia).
4. True (because there is at least one cat that is a mammal; in fact every cat is a mammal).
5. False (because we know that it is not possible to find at least cat that isn't a mammal)

EXAMPLE 2.1.1 #1 above illustrates the following fundamental fact:

In order for a statement of the form "All A are B" to be false, we must be able to demonstrate that there is at least one member of category A that isn't a member of category B. This is equivalent to demonstrating that A is not a subset of B. This means that a statement of the form "All A are B" is true even in the odd case where category A has no members.

EXAMPLES 1.4.1 #4 and #5 illustrate the following fundamental fact:

Although the statements "Some are..." and "Some aren't..." sound similar, they do not mean the same thing.

EXAMPLE 2.1.1*

True story: in the spring of 1999, a man in Tampa, Florida was diagnosed with stomach cancer. He underwent surgery to have the cancer removed. During this procedure, the surgical team discovered that in fact there was no cancer after all; the original diagnosis was incorrect. After the surgery, the physicians told the patient "All of the cancer has been removed." Did the physicians lie?

NEGATIONS

If p is a statement, the **negation** of p is another statement that is exactly the opposite of p . The negation of a statement p is denoted $\sim p$ ("**not p**").

A statement p and its negation $\sim p$ will always have opposite truth values; it is impossible to conceive of a situation in which a statement and its negation will have the same truth value.

EXAMPLE

Let p be the statement "Today is Saturday."

Then $\sim p$ is the statement "Today is not Saturday."

On any given day, if p is true then $\sim p$ will be false; if p is false, then $\sim p$ will be true.

It is impossible to conceive of a situation in which p and $\sim p$ are simultaneously true.

It is impossible to conceive of a situation in which p and $\sim p$ are simultaneously false.

NEGATIONS OF QUANTIFIED STATEMENTS

Fact: "None" is the opposite of "at least one."

For example: The negation of "Some dogs are poodles" is "No dogs are poodles."

Notice that "Some dogs are poodles" is a statement that is true according to our everyday experience, and "No dogs are poodles" is a statement that is false according to our everyday experience.

In general:

The negation of "Some A are B" is "No A are (is) B."

(Note: this can also be phrased "All A are the opposite of B," although this construction sometimes sounds ambiguous.)

EXAMPLE 2.1.2

Write the negation of "Some used cars are reliable."

Fact: "Some aren't" is the opposite of "all are."

For example, the negation of "All goats are mammals" is "Some goats aren't mammals."

Notice that "All goats are mammals" is a statement that is true according to our everyday experience, while "Some goats aren't mammals" is a statement that is false according to our everyday experience.

In fact, it is logically impossible to imagine a situation in which those two statements have the same truth value.

In general, the negation of "All A are B" is "Some A aren't B."

EXAMPLE 2.1.3

Write the negation of "All acute angles are less than 90° in measure."

EXAMPLE 2.1.4

Write the negation of "No triangles are quadrilaterals."

WORLD WIDE WEB NOTE

For practice in recognizing the negations of quantified statements, visit the companion website and try The QUANTIFIER-ER.

LOGICAL CONNECTIVES

*The words "and" "or" "but" "if...then" are examples of **logical connectives**. They are words that can be used to connect two or more simple statements to form a more complicated **compound statement**.*

Examples of compound statements:

"I am taking a math class **but** I'm not a math major."

"**If** I pass MGF1106 **and** I pass MGF1107 **then** my liberal studies math requirement will be fulfilled."

EQUIVALENT STATEMENTS

Any two statements p and q are **logically equivalent** if they have exactly the same meaning. This means that p and q will always have the same truth value, in any conceivable situation.

If p and q are equivalent statements, then it is logically impossible to imagine a situation in which the two statements would have differing truth values.

Examples:

"Today I have math class and today is Saturday" is equivalent to "Today is Saturday and today I have math class."

This equivalency follows simply from our everyday understanding of the meaning of the word "and."

"This and that" means the same as "That and this."

Likewise, "I have a dog or I have a cat" is equivalent to "I have a cat or I have a dog"
This equivalency follows simply from our everyday understanding of the meaning of the word "or."

"This or that" means the same as "That or this."

Logical equivalence is denoted by this symbol: \equiv

Referring back to examples 1.4.1 #4 and #5 we saw that the statement "Some cats are mammals" was true, while the statement "Some cats aren't mammals" was false. This means that those two statements are NOT equivalent.

The pair of statements cited above illustrate this general fact:

"Some A are B" is not equivalent to "Some A aren't B."

THE CONJUNCTION AND THE DISJUNCTION

THE CONJUNCTION

If p , q are statements, their **conjunction** is the statement " p and q ."

It is denoted: $p \wedge q$

For example, let p be the statement "I have a dime" and let q be the statement "I have a nickel." Then $p \wedge q$ is the statement "I have a dime **and** I have a nickel."

In general, in order for any statement of the form " $p \wedge q$ " to be true, both p and q must be true.

Example: "Tallahassee is in Florida and Orlando is in Georgia" is a false statement.

MORE ON THE CONJUNCTION

The word **but** is also a conjunction; it is sometimes used to precede a negative phrase.

Example: "I've fallen **and** I can't get up" means the same as "I've fallen **but** I can't get up."

In either case, if p is "I've fallen" and q is "I *can* get up" the conjunction above is symbolized as $p \wedge \sim q$.

THE DISJUNCTION

If p , q are statements, their *disjunction* is the statement " p or q ."

It is denoted: $p \vee q$.

For example, let p be the statement "Today is Tuesday" and let q be the statement " $1 + 1 = 2$." In that case, $p \vee q$ is the statement "Today is Tuesday **or** $1 + 1 = 2$."

In general, in order for a statement of the form $p \vee q$ to be true, at least one of its two parts must be true. The only time a disjunction is false is when both parts (both "components") are false.

The statement "Today is Tuesday **or** $1 + 1 = 2$ " is TRUE.

EQUIVALENCIES FOR THE CONJUNCTION ("AND") AND THE DISJUNCTION ("OR")

As we observed earlier, according to our everyday usage of the words "and" and "or" we have the following equivalencies:

1. "p and q" is equivalent to "q and p"

$$p \wedge q \equiv q \wedge p$$

2. "p or q" is equivalent to "q or p"

$$p \vee q \equiv q \vee p$$

For example, "I have a dime or I have a nickel" equivalent to "I have a nickel or I have a dime."

Likewise, "It is raining and it isn't snowing" is equivalent to "It isn't snowing and it is raining."

EXAMPLE 2.1.6

Suppose p and q are true statements, while r is a false statement. Determine the truth value of

1. $\sim q \vee r$

2. $\sim(r \wedge q)$

3. $\sim[(p \wedge \sim r) \vee q]$

Solution for EXAMPLE 2.1.6 #2

We are given the statement $\sim(r \wedge q)$ where q is true, r is false. Substitute the value T for the variable q , and the value F for the variable r :

$$\sim(F \wedge T)$$

Now, evaluate the expression inside the parentheses.

A conjunction is only true in the case where both of its components are true, so in this case the expression inside the parentheses is false.

Now the statement simplifies to: $\sim(F)$

The negation of false means the opposite of false, which is true.

So, the truth value of the given statement, under the given conditions, is TRUE.

WORLD WIDE WEB NOTE

For practice problems involving the truth values of symbolic statements, visit the companion website and try THE LOGICIZER

TRUTH TABLES

*A **truth table** is a device that allows us to analyze and compare compound logic statements.*

Consider the symbolic statement $p \vee \sim q$.

Whether this statement is true or false depends upon whether its variable parts are true or false, as well as on the behavior of the “or” connective and the “negation” operator.

Later, we will make a truth table for this statement.

A truth table for this statement will take into account every possible combination of the variables being true or false, and show the truth value of the compound statement in each case.

EXAMPLE 2.1.7

As an introduction, we will make truth tables for these two statements

1. $p \wedge q$
2. $p \vee q$

Solution to EXAMPLE 2.1.7 #1

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Note that in this truth table there is only one row in which the statement $p \wedge q$ is true. This is the row where p is true and q is true. This conforms to our earlier observation that the only situation in which a conjunction is true is the case in which both of its component statements are true.

Solution to EXAMPLE 2.1.7 #2

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note that in this truth table there is only one row in which the statement $p \vee q$ is false. This is the row where p is false and q is false. This conforms to our earlier observation that the only situation in which a disjunction is false is the case in which both of its component statements are false.

THE BASIC RULES FOR CONSTRUCTING A TRUTH TABLE FOR A COMPOUND STATEMENT

1. The number of **rows** in the truth table depends upon the number of basic variables in the compound statement. To determine the number of rows required, count the number of basic variables in the statement, but **don't** re-count multiple occurrences of a variable.

1 variable---2 rows

2 variables--4 rows

3 variables--8 rows

4 variables--16 rows and so forth.

2. The number of **columns** in a truth table depends upon the number of logical connectives in the statement. The following guidelines are usually reliable.

A. There will be one column for each basic variable; and

B. To determine the number of other columns, count the number of logical connectives in the statement; **do** re-count multiple occurrences of the same connective. The “~” symbol counts as a logical connective.

In addition to the columns for each basic variable, there will usually be one column for each occurrence of a logical connective.

3. The beginning columns are filled in so as to take into account every possible combination of the basic variables being true or false. Each **row** represents one of the possible combinations.

4. In order to fill in any other column in the truth table, you must **refer to a previous column or columns**.

EXAMPLE 2.1.7 #3

Make a truth table for the following statement: $(p \wedge \sim q) \vee \sim p$

EXAMPLE 2.1.8

Make truth tables for the following statements:

1. $p \vee \sim q$
2. $q \vee \sim(\sim p \wedge q)$

Solution to EXAMPLE 2.1.8 #1**Step 1: Determine the number of rows required.**

Since the statement contains two basic variables, the truth table will require four rows, because $2^2 = 4$.

Step 2: Determine the number of columns required.

There will be one column for each basic variable, and one column for each occurrence of a logical connective in the statement $p \vee \sim q$. This means that we will have a total of **four** columns.

Step 3: Begin filling in the columns.

The first two columns represent the basic variables p, q .

We label them accordingly, and fill them in in such a way that each row takes into account a different combination of truth values for these basic variables. The configuration shown below is standard.

p	q		
T	T		
T	F		
F	T		
F	F		

STEP 4: Label the remaining columns, bearing in mind which simpler components are required in order to construct the statement $p \vee \sim q$.

In order to construct the statement $p \vee \sim q$, we need a column for p and a column for $\sim q$. The truth table already has a column for p and a column for q , so we now label the next column $\sim q$. We fill in this column by referring to the values in the column for q ; every entry in the column $\sim q$ will be the opposite of the corresponding entry in the column for q :

p	q	$\sim q$	
T	T	F	
T	F	T	
F	T	F	
F	F	T	

Now that we have a column for p as well as a column for $\sim q$, we can combine them to construct a column for $p \vee \sim q$. We fill in this column by referring to the columns for p and for $\sim q$, and bearing in mind the behavior of the "or" connective: $p \vee \sim q$ will be **TRUE** in any row where the column for p is true, or the column for $\sim q$ is true, or both; $p \vee \sim q$ will be **FALSE** only in a row where the p and $\sim q$ are both false.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

This complete truth table shows the behavior of the statement in every conceivable situation. As we will see later, it can be used to compare the statement $p \vee \sim q$ with other compound statements, and to identify other, different-looking statements that are equivalent to $p \vee \sim q$.

WORLD WIDE WEB NOTE

For practice problems involving truth tables, visit the companion website and try **THE TRUTH TABLER**

TAUTOLOGIES

Referring to the truth table for the statement $q \vee \sim (\sim p \wedge q)$ in the previous example: notice that the column for that statement shows only "true." This means that it is never possible for a statement of the form $q \vee \sim (\sim p \wedge q)$ to be false.

A **tautology** is a statement that cannot possibly be false, due to its logical structure (its **syntax**).

The statement $q \vee \sim (\sim p \wedge q)$ is an example of a tautology.

EXAMPLE 2.1.9

Is the statement $(p \vee r) \vee \sim (p \wedge q)$ a tautology?

We can answer this question by making a truth table.

EXAMPLE 2.1.9A

Is the statement $(p \vee \sim q) \vee (\sim p \wedge q)$ a tautology?

We can answer this question by making a truth table.

EXAMPLE 2.1.10

Compare the truth table column for $p \vee \sim q$ (EXAMPLE 2.1.8 #1) to the column for $\sim(\sim p \wedge q)$ (this is the second column from the right in the solution for EXAMPLE 2.1.8 #2).

Solution to EXAMPLE 2.1.10

In making the comparison, we see that the two columns are identical: each column has "T" in the first row, "T" in the second row, "F" in the third row, and "T" in the fourth row. *When two statements have identical truth table columns, the statements are equivalent.*

USING TRUTH TABLES TO TEST FOR LOGICAL EQUIVALENCY

To determine if two statements are equivalent, make a truth table having a column for each statement. If the columns are identical, then the statements are equivalent.

From EXAMPLE 2.1.10, we see that $p \vee \sim q \equiv \sim(\sim p \wedge q)$

EXAMPLE 2.1.12

Use the result from EXAMPLE 2.1.10 to write a statement that is equivalent to "It is not the case that both I won't order a taco and I will order a burrito."

Solution to EXAMPLE 2.1.12

If p represents "I will order a taco" and q represents "I will order a burrito" then the statement "It is not the case that both I won't order a taco and I will order a burrito" is symbolized as $\sim(\sim p \wedge q)$.

The result from EXAMPLE 2.1.10 tells us that $\sim(\sim p \wedge q)$ is equivalent to "I will order a taco or I won't order a burrito."

EXAMPLE 2.1.14

1. Use a truth table to determine whether $\sim(p \wedge q)$ is equivalent to $\sim p \wedge \sim q$.
2. Use a truth table to determine whether $\sim(p \wedge q)$ is equivalent to $\sim p \vee \sim q$.
3. Use a truth table to determine whether $\sim(p \vee q)$ is equivalent to $\sim p \vee \sim q$.
4. Use a truth table to determine whether $\sim(p \vee q)$ is equivalent to $\sim p \wedge \sim q$.

EXAMPLE 2.1.15

Complete the following truth table.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$\sim(\sim p \vee q)$	$\sim p \vee (p \wedge \sim q)$	$q \wedge \sim(\sim p \vee q)$
T	T	F	F					
T	F	F	T					
F	T	T	F					
F	F	T	T					

Identify any equivalencies or tautologies.

EXAMPLE 2.1.16

Make a truth table for $\sim(p \wedge r) \vee q$

DeMORGAN'S LAWS

From EXAMPLE 2.1.12 #2 and 1.4.12 #4 we have the following rules of logical equivalency:

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

These two rules are called DeMorgan's Laws for Logic. Compare them with DeMorgan's Laws for Set Mathematics (see Unit 1 Module 2). Although they are written as equivalencies, in fact they tell us how to write the negation of a conjunction or disjunction. In words:

THE NEGATION OF A CONJUNCTION

The negation of "p and q" is "not p or not q."

THE NEGATION OF A DISJUNCTION

The negation of "p or q" is "not p and not q."

EXAMPLE 2.1.17

Use DeMorgan's Laws to write the negation of each statement:

1. I want a car and a motorcycle.
2. My cat stays outside or it makes a mess.
3. I've fallen and I can't get up.
4. You study or you don't get a good grade.

Solutions to EXAMPLE 2.1.17

1. I don't want a car or I don't want a motorcycle.
2. My cat doesn't stay outside and it doesn't make a mess.
3. I haven't fallen or I can get up.
4. You don't study and you get a good grade.

EXAMPLE 2.1.18

1. Select the statement that is the negation of "Today is Monday and it isn't raining."
 - A. Today isn't Monday and it isn't raining.
 - B. Today isn't Monday or it isn't raining.
 - C. Today isn't Monday or it is raining.
 - D. Today isn't Monday and it is raining.
 - E. Today is Friday and it is snowing.

2. Select the statement that is the negation of "I'm careful or I make mistakes."
 - A. I'm not careful and I don't make mistakes.
 - B. I'm not careful or I don't make mistakes.
 - C. I'm not careful and I make mistakes.
 - D. I'm not careful or I make mistakes.
 - E. I never make mistakes.

EXAMPLE 2.1.19

1. Select the statement that is the negation of "I walk or I chew gum."
 - A. I don't walk and I chew gum.
 - B. I don't walk or I chew gum.
 - C. I don't walk and I don't chew gum.
 - D. I don't walk or I don't chew gum.
 - E. I walk until I step on chewed gum.
2. Select the statement that is the negation of "I'm mad as heck and I'm not going to take it anymore."
 - A. I'm not mad as heck and I'm not going to take it anymore.
 - B. I'm not mad as heck or I'm not going to take it anymore.
 - C. I'm not mad as heck and I am going to take it anymore.
 - D. I'm not mad as heck or I am going to take it anymore.

3. Select the statement that is the negation of "All of the businesses are closed."

- A. Some of the businesses are closed.
- B. Some of the businesses are not closed.
- C. None of the businesses are closed.
- D. All of the businesses are open.
- E. All of my clothes are businesslike.

4. Select the statement that is the negation of the statement "Some pilots are pirates."

- A. All pilots are pirates.
- B. No pilots are pirates.
- C. Some pilots are not pirates.
- D. All pirates are pilots.

EXAMPLE 2.1.20

*'But wait a bit,' the Oysters cried,
'Before we have our chat;
For some of us are out of breath,
And all of us are fat!'
"No hurry!" said the Carpenter.
They thanked him much for that.*

Select the statement that is the negation of "Some of us are out of breath, and all of us are fat."

- A. Some of us aren't out of breath or none of us is fat.
- B. Some of us aren't out of breath and none of us is fat.
- C. None of us is out of breath and some of us aren't fat.
- D. None of us is out of breath or some of us aren't fat.

ANOTHER EQUIVALENCY FOR THE "OR" STATEMENT

Consider this disjunction: "You will behave or you won't get a reward."

Can you think of another statement that conveys exactly the same meaning without using the word "or"?

Fact: any statement of the form "p or q" can be written equivalently in the form "If not p, then q." This is denoted $\neg p \rightarrow q$ and will be discussed in detail in Module 1.5. Convince yourself that the warning "You will behave or you won't get a reward" conveys the same information as the warning "If you don't behave, then you won't get a reward."

A SUMMARY OF FACTS ABOUT THE CONJUNCTION AND THE DISJUNCTION.

Equivalencies

Statement	Equivalent form
p and q	q and p
p or q	q or p
p or q	If not p then q

Negations (DeMorgan's Laws)

Statement	Negation
p and q	not p or not q
p or q	not p and not q

EXAMPLE 2.1.21

Select the statement that is logically equivalent to "I'm careful or I make mistakes."

- A. I'm careful and I make mistakes.
- B. I make mistakes and I'm careful.
- C. If I'm not careful, then I make mistakes.
- D. None of these.

DIAGRAMMING CATEGORICAL STATEMENTS

Diagramming is a technique involving the use of Venn diagrams to study the relationships between multiple categorical statements. It is particularly useful in situations involving two or three categories, and two or more categorical statements.

When diagramming a universal statement, such as "All dogs are mammals" or "No dogs are cats," we use **shading** to indicate that a certain region of the diagram must contain **no elements**.

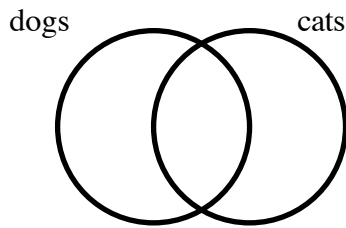
When diagramming an existential statement, such as "Some dogs are poodles" or "Some dogs aren't scavengers," we use an "X" to indicate that a certain region of the diagram

must contain at least one element. If it is not certain which of two regions must contain an element, then we place an “X” on the border between the two regions.

If a region of a marked diagram contains no marks, then it is uncertain whether that region contains any elements.

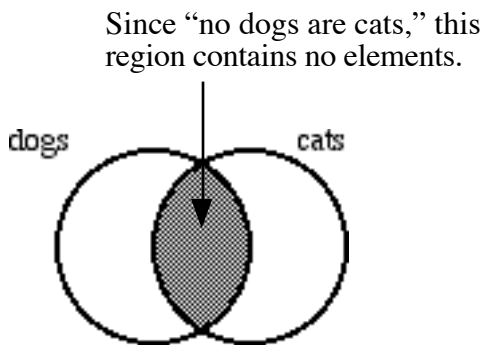
EXAMPLE 2.1.22

Mark the diagram according to the statement “No dogs are cats.”



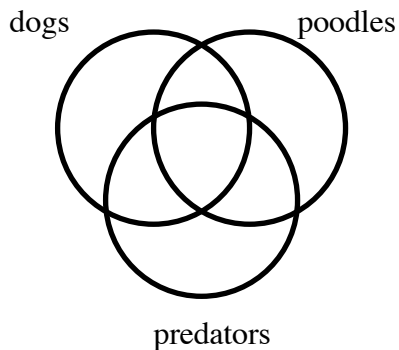
SOLUTION

If “no dogs are cats,” then the part of the diagram where those two circles intersect must contain no elements. We use **shading** to indicate that a region of the diagram must contain no elements.



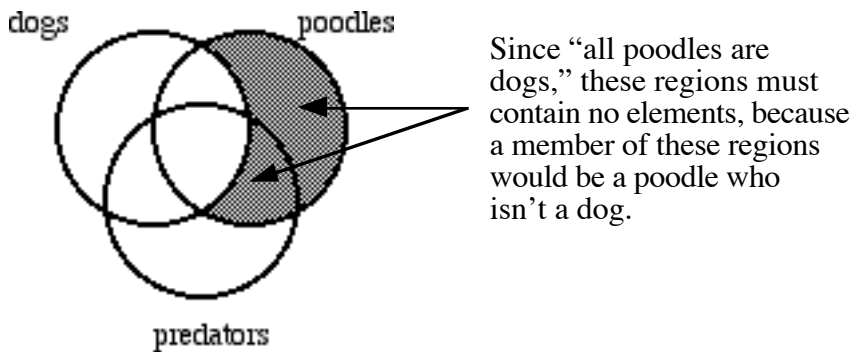
EXAMPLE 2.1.23

Mark the diagram according to both of these statements: “All poodles are dogs” and “Some dogs are predators.”



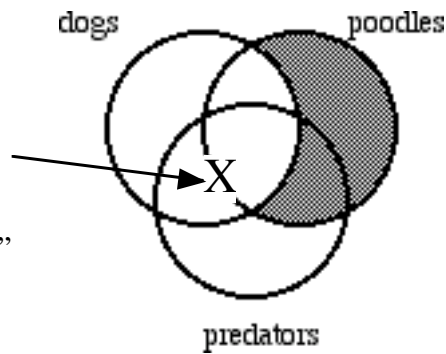
SOLUTION

First, the universal statement “All poodles are dogs” asserts, indirectly, that a certain region of the diagram must contain no elements (and this is the only thing that such a statement asserts; in particular, a universal statement never asserts the existence of any elements).



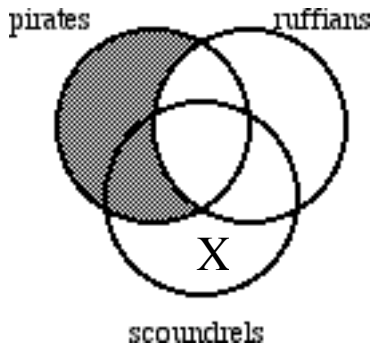
Next, continue marking the diagram according to the existential statement “Some dogs are predators.” This means that there must be at least one element in the part of the diagram that is where “dogs” and “predators” intersect.

Since “some dogs are predators,” there must be at least one element in the intersection of “dogs” and “predators.” Since it isn’t certain which of these two regions must contain the elements, we place the “X” on the boundary.



EXAMPLE 2.1.24

Suppose the marked diagram below conveys information about relationships between pirates, ruffians and scoundrels. We use shading to indicate that a region contains no elements. An “X” in a region indicates the existence of at least one element; an “X” on the boundary between two regions indicates that the union of those two regions contains at least one element. If a region is unmarked, then whether that region contains any elements is uncertain. Select the choice that must be true according to the marked diagram.



- A. No pirates are ruffians and some ruffians aren't scoundrels.
- B. No pirates are ruffians and some scoundrels aren't ruffians.
- C. All pirates are ruffians and some ruffians aren't scoundrels.
- D. All pirates are ruffians and some scoundrels aren't ruffians.
- E. None of these is correct.

PRACTICE EXERCISES

1. Suppose p is the statement 'You need a credit card' and q is the statement 'I have a nickel.'

Select the correct statement corresponding to the symbols $\sim(p \vee q)$.

- A. You don't need a credit card and I have a nickel.
- B. It is not the case that either you need a credit card or I have a nickel.
- C. You don't need a credit card or I have a nickel.
- D. None of these.

2. Suppose p is the statement 'There are 1,000 meters in one kilometer' and q is the statement 'You will give me a cake.' Select the correct symbolization for the statement 'There are 1,000 meters in one kilometer or you will not give me a cake'.

- A. $\sim(p \wedge q)$
- B. $p \wedge \sim q$
- C. $p \vee \sim q$
- D. None of these

3. Suppose p is the statement 'There are 1,000 meters in one kilometer' and q is the statement 'You will order a burrito.' Select the correct symbolization for the statement 'There are not 1,000 meters in one kilometer and you won't order a burrito'.

- A. $\sim p \wedge \sim q$
- B. $\sim(p \wedge q)$
- C. $\sim p \vee \sim q$
- D. None of these

4. Suppose p is the statement 'I play softball' and q is the statement 'The moon is 250,000 miles from Earth.' Select the correct statement corresponding to the symbols $\sim p \wedge q$.

- A. I don't play softball and the moon is 250,000 miles from Earth.
- B. It is not the case that either I play softball or the moon is 250,000 miles from Earth.
- C. I don't play softball or the moon is 250,000 miles from Earth.
- D. It is not the case that both I play softball and the moon is 250,000 miles from Earth.

5. Suppose p is false, q is false, s is true. Find the truth value of $(s \vee p) \wedge (q \wedge \sim s)$

6. Suppose p is true, q is true, r is false, s is false. Find the truth value of $(s \vee p) \wedge (\sim r \vee \sim s)$

7. Suppose p is true, q is true, s is false. Find the truth value of $(\sim s \vee p) \vee (q \wedge \sim s)$

8. Suppose p is false, s is false, r is true. Find the truth value of $\sim[(s \wedge p) \vee \sim r]$

9. Suppose p is false, q is true, s is true. Find the truth value of $(p \wedge \sim q) \vee \sim s$

10. Suppose p is false, q is true, r is false. Find the truth value of $(p \vee \sim q) \vee r$

11. Suppose p is true, q is true, r is true, s is false. Find the truth value of $(\sim p \vee s) \vee (s \wedge r)$

12-17: Make a truth table for the given expression.

12. $(\sim p \wedge q) \vee (p \wedge \sim q)$ 13. $(p \wedge \sim q) \vee r$ 14. $\sim[(p \wedge \sim q) \vee \sim p]$
15. $(p \vee q) \wedge \sim(\sim q \wedge r)$ 16. $(\sim p \wedge q) \vee (\sim p \vee q)$ 17. $\sim[(p \vee q) \wedge \sim q]$

18 – 21: In each case, decide whether the statement is true or false.

18. True or false: $(\sim p \wedge q) \vee (\sim p \vee q) \equiv \sim[(p \vee q) \wedge \sim q]$

Hint: refer to the answers to #16 and #17 above.

19. True or false: $(\sim p \wedge q) \vee (p \wedge \sim q)$ is a tautology. *Hint: refer to the answer to #12 above.*

20. True or false: $(\sim p \wedge q) \vee (\sim p \vee q) \equiv (\sim p \vee q)$ *Hint: refer to the answer to #16 above.*

21. True or false: $(p \vee q) \wedge \sim(\sim q \wedge r) \equiv (p \wedge \sim q) \vee r$

Hint: refer to the answers to #15 and #13 above

22. Select the statement that is the **negation** of “All summer days are muggy.”

- A. All muggy days are summer. B. Some summer days are muggy.
C. Some summer days are not muggy. D. No summer days are muggy.

23. Select the statement that is the **negation** of “Some weasels are cuddly.”

- A. No weasels are cuddly. B. All weasels are cuddly.
C. Some weasels are not cuddly. D. All cuttlefish are weasely.

24. Select the statement that is the **negation** of

“Coach Spurrier is charming and Coach Spurrier is modest.”

- A. Coach Spurrier is not charming and Coach Spurrier is not modest.
B. Coach Spurrier is not charming or Coach Spurrier is not modest.
C. Coach Spurrier is not charming and Coach Spurrier is modest.
D. Let’s get serious for a minute.

25. Select the statement that is the **negation** of

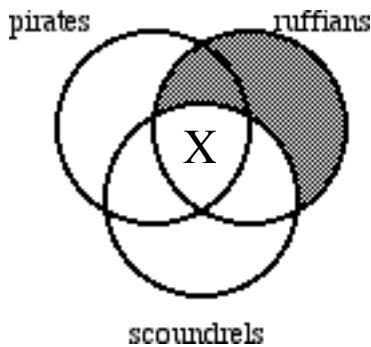
“The speed limit is 55 and granny is driving 35.”

- A. The speed limit is not 55 or granny is not driving 35.
B. The speed limit is not 55 and granny is not driving 35
C. The speed limit is not 55 or granny is driving 35.
D. The speed limit is not 55 and granny is driving 35.
E. Counseling for road rage is available at 1 - 900 - calmdown.

26. Select the statement that is the **negation** of “All circus clowns are undignified.”
- A. All circus clowns are dignified. B. All cirrus clouds are indistinguishable.
 C. Some circus clowns are dignified. D. No circus clowns are dignified.

27. Select the statement that is the **negation** of “You wear matching socks to the interview or you don’t get hired.”
- A. You don’t wear matching socks to the interview or you get hired.
 B. You don’t wear matching socks to the interview and you get hired.
 C. You don’t wear matching socks to the interview and you don’t get hired.
 D. If you don’t wear matching socks to the interview, then you don’t get hired.

28. Suppose the marked diagram below conveys information about relationships between pirates, ruffians and scoundrels. We use shading to indicate that a region contains no elements. An “X” in a region indicates the existence of at least one element; an “X” on the boundary between two regions indicates that the union of those two regions contains at least one element. If a region is unmarked, then whether that region contains any elements is uncertain. Select the choice that must be true according to the marked diagram.



- A. No scoundrels are ruffians and some pirates aren’t scoundrels.
 B. All ruffians are scoundrels and some scoundrels aren’t pirates.
 C. No pirates are scoundrels and some ruffians are scoundrels.
 D. All ruffians are scoundrels and some pirates are ruffians.
 E. None of these is correct.

ANSWERS TO LINKED EXAMPLES

EXAMPLE 2.1.1* Although the physicians could have been more candid, from the point of view of logic they did not lie.

EXAMPLE 2.1.2 No used car is reliable.

EXAMPLE 2.1.3 Some acute angles are not less than 90° in measure.

EXAMPLE 2.1.4 Some triangles are quadrilaterals.

EXAMPLE 2.1.6 1. F 3. F

EXAMPLE 2.1.7 #3

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \vee \sim p$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

EXAMPLE 2.1.8 #2

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$	$q \vee \sim(\sim p \wedge q)$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

EXAMPLE 2.1.9

p	q	r	$p \wedge q$	$\sim(p \wedge q)$	$p \vee r$	$(p \vee r) \vee \sim(p \wedge q)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	F	T

The truth table shows that the statement $(p \vee r) \vee \sim(p \wedge q)$ is a tautology.

EXAMPLE 2.1.9A The statement $(p \vee \sim q) \vee (\sim p \wedge q)$ is not a tautology.

EXAMPLE 2.1.14

1. Not equivalent 2. Equivalent 3. Not equivalent 4. Equivalent

EXAMPLE 2.1.15

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$\sim(\sim p \vee q)$	$\sim p \vee (p \wedge \sim q)$	$q \wedge \sim(\sim p \vee q)$
T	T	F	F	F	T	F	F	F
T	F	F	T	T	F	T	T	F
F	T	T	F	F	T	F	T	F
F	F	T	T	F	T	F	T	F

The truth table shows no tautologies. The truth table shows the following equivalency:

$$p \wedge \sim q \equiv \sim(\sim p \vee q)$$

EXAMPLE 2.1.18 1. C 2. A

EXAMPLE 2.1.19 1. C 2. D 3. B 4. B

EXAMPLE 2.1.20 D

EXAMPLE 2.1.21 C

EXAMPLE 2.1.24 D

ANSWERS TO PRACTICE EXERCISES

1. B 2. C 3. A 4. A

5. Suppose p is false, q is false, s is true. Then $(s \vee p) \wedge (q \wedge \sim s)$ is F.

6. Suppose p is true, q is true, r is false, s is false. Then $(s \vee p) \wedge (\sim r \vee \sim s)$ is T.

7. Suppose p is true, q is true, s is false. Then $(\sim s \vee p) \vee (q \wedge \sim s)$ is T.

8. Suppose p is false, s is false, r is true. Then $\sim[(s \wedge p) \vee \sim r]$ is T.

9. Suppose p is false, q is true, s is true. Then $(p \wedge \sim q) \vee \sim s$ is F.

10. Suppose p is false, q is true, r is false. Then $(p \vee \sim q) \vee r$ is F.

11. Suppose p is true, q is true, r is true, s is false. Then $(\sim p \vee s) \vee (s \wedge r)$ is F.

12. $(\sim p \wedge q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \wedge \sim q$	$(\sim p \wedge q) \vee (p \wedge \sim q)$
T	T	F	F	F	F	F
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	F	F	F

13. $(p \wedge \sim q) \vee r$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \wedge \sim q$	$(p \wedge \sim q) \vee r$
T	T	T	F	F	F	F	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	F	F	T
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	T
F	F	F	T	T	T	F	F

14. $\sim[(p \wedge \sim q) \vee \sim p]$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \vee (\sim p)$	$\sim[(p \wedge \sim q) \vee (\sim p)]$
T	T	F	F	F	F	T
T	F	F	T	T	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

15. $(p \vee q) \wedge \sim(\sim q \wedge r)$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \vee q$	$\sim q \wedge r$	$\sim(\sim q \wedge r)$	$(p \vee q) \wedge \sim(\sim q \wedge r)$
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	T	T	F	T	T
T	F	T	F	T	F	T	T	F	F
T	F	F	F	T	T	T	F	T	T
F	T	T	T	F	F	T	F	T	T
F	T	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	F
F	F	F	T	T	T	F	F	T	F

16. $(\sim p \wedge q) \vee (\sim p \vee q)$

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim p \vee q$	$(\sim p \wedge q) \vee (\sim p \vee q)$
T	T	F	F	F	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	F	T	T

17. $\sim[(p \vee q) \wedge \sim q]$

p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee q) \wedge (\sim q)$	$\sim[(p \vee q) \wedge (\sim q)]$
T	T	F	F	T	F	T
T	F	F	T	T	T	F
F	T	T	F	T	F	T
F	F	T	T	F	F	T

18. True

19. False

20. True

21. False

22. C

23. A

24. B

25. A

26. C

27. B

28. D