PART 3 MODULE 3
CLASSICAL PROBABILITY, STATISTICAL PROBABILITY, ODDS

PROBABILITY

Classical or **theoretical** definitions:
Let S be the set of all equally likely outcomes to a random experiment.
(S is called the *sample space* for the experiment.)
Let E be some particular outcome or combination of outcomes to the experiment.
(E is called an *event*.)
The *probability of E* is denoted P(E).

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of outcomes favorable to E}}{\text{number of possible outcomes}}$$

EXAMPLE 3.3.1
Roll one die and observe the numerical result. Then S = {1, 2, 3, 4, 5, 6}.
Let E be the event that the die roll is a number greater than 4.
Then E = {5, 6}

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = .333\overline{3}$$

EXAMPLE 3.3.2
Referring to the earlier example (from Unit 3 Module 3) concerning the ***National Requirer***. What is the probability that a randomly selected story will be about Elvis?

EXAMPLE 3.3.2 solution
In solving that problem (EXAMBLE 3.3.14) we saw that there were 21 possible storylines. Of those 21 possible story lines, 12 were about Elvis. Thus, if one story line is randomly selected or generated, the probability that it is about Elvis is 12/21, or roughly .571.

EXAMPLE 3.3.3
An office employs seven women and five men. One employee will be randomly selected to receive a free lunch with the boss. What is the probability that the selected employee will be a woman?

EXAMPLE 3.3.4
An office employs seven women and five men. Two employees will be randomly selected for drug screening. What is the probability that both employees will be men?

EXAMPLE 3.3.5

Roll one die and observe the numerical result. Then S = {1, 2, 3, 4, 5, 6}.
Let E be the event that the die roll is a number greater than 4.
We know that $P(E) = \frac{2}{6} = .33\overline{3}$

What about the probability that E doesn't occur?

We denote this as $P(E')$

Then $E' = \{1,2,3,4\}$, so $P(E') = \frac{4}{6} = .66\overline{6}$

Note that $\mathbf{P(E') = 1 - P(E)}$

This relationship (the **Complements Rule**) will hold for any event E:
"The probability that an event *doesn't* occur is 1 minus the probability that the event *does* occur."

EXAMPLE 3.3.6

Again, the experiment consists in rolling one die.
Let F be the event that the die roll is a number less than 7.
Then F = {1, 2, 3, 4, 5, 6}
So $P(F) = \frac{n(F)}{n(S)} = \frac{6}{6} = 1$

If an event is certain to occur, then its probability is 1.
Probabilities are never greater than 1.

EXAMPLE 3.3.7

Let G be the event that the die roll is "Elephant."

Then G = { }

So $P(G) = \frac{n(G)}{n(S)} = \frac{0}{6} = 0$

If an event is impossible, then its probability is 0.

Probabilities are never less than 0.

We have the following scale:

For any event E in any experiment, $\mathbf{0 \le P(E) \le 1}$

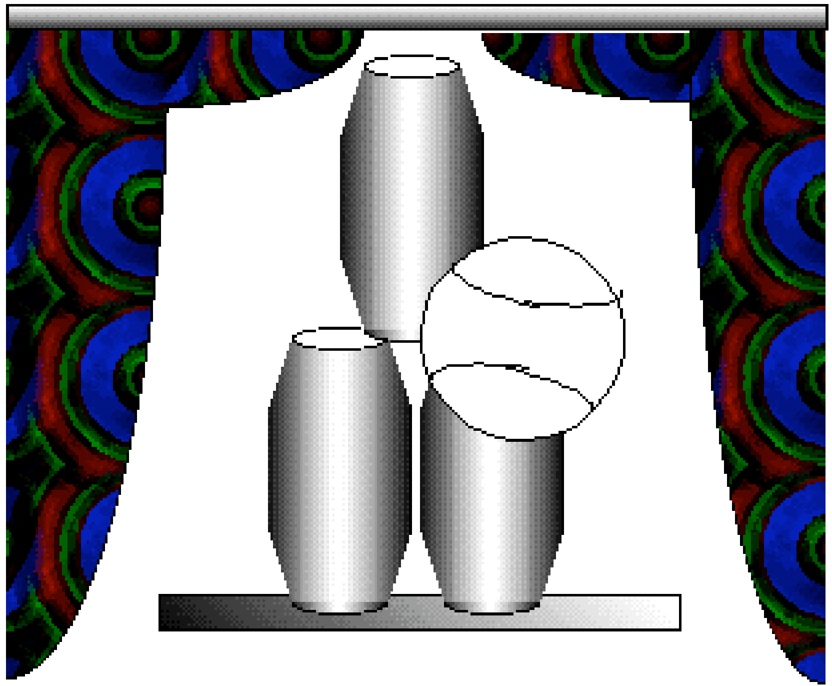
EXAMPLE 3.3.8
A jar contains a penny, a nickel, a dime, a quarter, and a half-dollar. Two coins are randomly selected (without replacement) and their monetary sum is determined.
1. What is the probability that their monetary sum will be 55¢?
A. 1/25 B. 1/32 C. 1/9 D. 1/10

2. What is the probability that the monetary sum will be 48¢?
A. 1/10 B. 1/9 C. 1/32 D. 0

EXAMPLE 3.3.9
What is the probability of winning the Florida Lotto with one ticket?

EMPIRICAL OR STATISTICAL PROBABILITY
EXAMPLE 3.3.10

A carnival game requires the contestant to throw a softball at a stack of three "bottles." If the pitched softball knocks over all three bottles, the contestant wins. We want to determine the probability that a randomly selected contestant will win (event E). How can this be done?



Note that the classical definition of probability does not apply in this case, because we can't break this experiment down into a set of equally likely outcomes.

For instance, one outcome of the experiment is the situation where no bottles are toppled. Another outcome is the case where 1 bottle is topples, another is the case where 2 bottles are topples, and yet another outcome is the case where all 3 bottles are toppled. However, we don't know that these outcomes are **equally likely.**

In cases where it is not possible or practical to analyze a probability experiment by breaking it down into equally likely outcomes, we can estimate probabilities by referring to accumulated results of repeated trials of the experiment. Such estimated probabilities are called **empirical** probabilities:

EMPIRICAL PROBABILITY

$$P(E) = \frac{\text{number of occurrences of E}}{\text{number of trials of the experiment}}$$

Suppose we observe the game for one weekend. Over this period of time, the game is played 582 times, with 32 winners. Based on this data, we find P(E).

$$P(E) = \frac{32}{582} \approx .055$$

The law of large numbers is a theorem in statistics that states that as the number of trials of the experiment increases, the observed empirical probability will get closer and closer to the theoretical probability.

We can also refer to population statistics to infer to probability of a characteristic distributed across a population. The **statistical probability** of an event E is the proportion of the population satisfying E.

EXAMPLE 3.3.11
For instance (this is authentic data), a recent (1999) study of bottled water, conducted by the Natural Resources Defense Council, revealed that:
40% of bottled water samples were merely tap water.
30% of bottled water samples were contaminated by such pollutants as arsenic and fecal bacteria.

Let E be the event "A randomly selected sample of bottled water is actually tap water."
Let F be the event "A randomly selected sample of bottled water is contaminated."

Then:
P(E) = 40% = .4
P(F) = 30% = .3

EXAMPLE 3.3.12
According to a recent article from the *New England Journal of Medical Stuff* ,

63% of cowboys suffer from saddle sores,
52% of cowboys suffer from bowed legs,
and 40% suffer from both saddle sores and bowed legs.

Let E be the event "A randomly selected cowboy has saddle sores."
Then P(E) = .63

Let F be the event "A randomly selected cowboy has bowed legs."
Then P(F) = .52

Likewise, P(cowboy has both conditions) = .4

ODDS
Odds are similar to probability, in that they involve a numerical method for describing the likelihoood of an event. Odds are defined differently however.

$$\text{Odds in favor of E} = \frac{n(E)}{n(E')} = \frac{\text{number of outcomes favorable to E}}{\text{number of outcomes unfavorable to E}}$$

Odds are usually stated as a ratio.

EXAMPLE 3.3.13
Let E be the event that the result of a die roll is a number greater than 4.
Then "the odds in favor of E" = 2/4 or "2 to 4" or "2:4"

PRACTICE EXERCISES

1 - 3: Here is the grade distribution for Professor de Sade's math class:

Grade	Frequency
A	0
B	2
C	4
D	28
F	44

If a student is randomly selected, what is the probability that he/she...
1. ... had a grade of C?
2. ...didn't have an F?
3. ...had an A?

4. Suppose that the FSU football team plays six home games this year, including games against Georgia Tech and Miami. If Gomer's uncle randomly picks two of his six tickets to give to Gomer, what is the probability that they will be for the Georgia Tech and Miami games?

5. So far this basketball season, Plato has attempted 82 free throws and has made 62 of them. What is the probability that he will make a given free throw?

6 - 8: A poll (1999) by the Colonial Williamsburg Foundation revealed the following (this data is authentic):

79% of Americans know that "Just Do It" is a Nike slogan.
47% know that the phrase "Life, Liberty and the Pursuit of Happiness" is found in the Declaration of Independence.
9% know that George Washington was a Revolutionary War general.

6. What is the probability that an American knows that the phrase "Life, Liberty and the Pursuit of Happiness" is found in the Declaration of Independence?
A. 47/79 B. 47/135 C. 47/88 D. 47/100

7. What is the probability that an American knows that George Washington was a Revolutionary War general?
A. .9 B. .1 C. .09 D. .01

8. What is the probability that an American does not know that "Just Do It" is a Nike slogan?
A. .79 B. .21 C. 7.9 D. 2.1

9. What are the odds in favor of a randomly selected American knowing that "Just Do It" is a Nike slogan?
A. 79:100 B. 21:100 C. 79:21 D. 21:79

10. A "combination" lock has a three-number "combination" where the numbers are chosen from the set {1, 2, 3, ... , 19, 20}.
What is the probability that the "combination" has no repeated numbers?
A. .00015 B. .75 C. .15 D. .855

11. Gomer is taking a 25-question multiple-choice test. He needs to get a 100% on this test in order to get a C- in the course. He knows the answers to 21 of the questions, but is clueless on the other four problems. If he just guesses at the other four problems, what is the probability that he will get a score of 100%? (For each multiple-choice problem there are four choices.)
A. .25 B. .0625 C. .004 D. .625

ANSWERS TO LINKED EXAMPLES
EXAMPLE 3.3.3 7/12 or .583
EXAMPLE 3.3.4 10/66 or .152
EXAMPLE 3.3.8 1. D 2. D
EXAMPLE 3.3.9 1/22,957,480

ANSWERS TO PRACTICE EXERCISES

1. 4/78 ≈ .051	**2.** 34/78 ≈ .436	**3.** 0	**4.** 1/15
5. 62/82 ≈ .756	**6.** D	**7.** C	**8.** B
9. C	**10.** D	**11.** C	