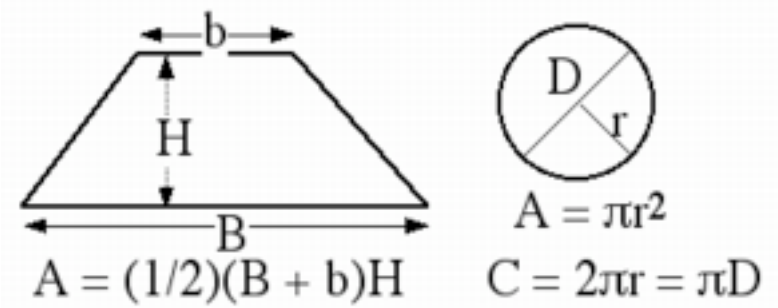
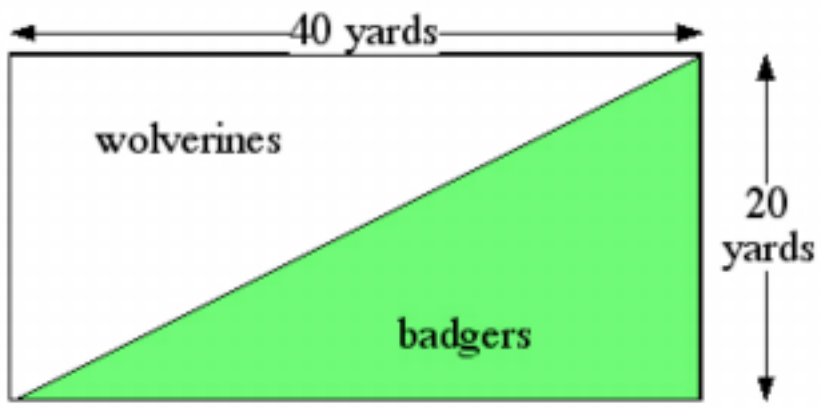
**PART 3 MODULE 8**
**PROBLEMS INVOLVING AREA**

We will be examining a variety of "real-world" problems that can be solved by referring to familiar facts from elementary geometry. These problems will usually require that we compute the area of one or more simple geometric figures, such as a rectangle, triangle, parallelogram, trapezoid or circle. The formulas for computing such areas are shown below.

**EXAMPLE 3.8.1**
The diagram below shows the rectangular pen in which Gomer confines his wolverines and badgers. As a special treat, Gomer is going to cover the badgers' area of the pen with Astroturf. Assuming that Astroturf costs $1.25 per square foot, how much with this project cost? A. $1500 B. $9000 C. $4500 D. $3000




**EXAMPLE 3.8.1 solution**
To solve this problem we need to find the area of the triangular region (in square feet) and multiply by the cost factor of $1.25 per square foot.
To find the area of the triangle, we use the formula A =(1/2)bh.
For this particular triangle, b = 40 yards and h = 20 yards.

Now, in the figure shown the measurements are expressed in yards, but we want to compute the triangle's area in square feet, not square yards. This suggests that we should convert units from yards to feet before using the formula for area. To convert from yards to feet we multiply by 3:

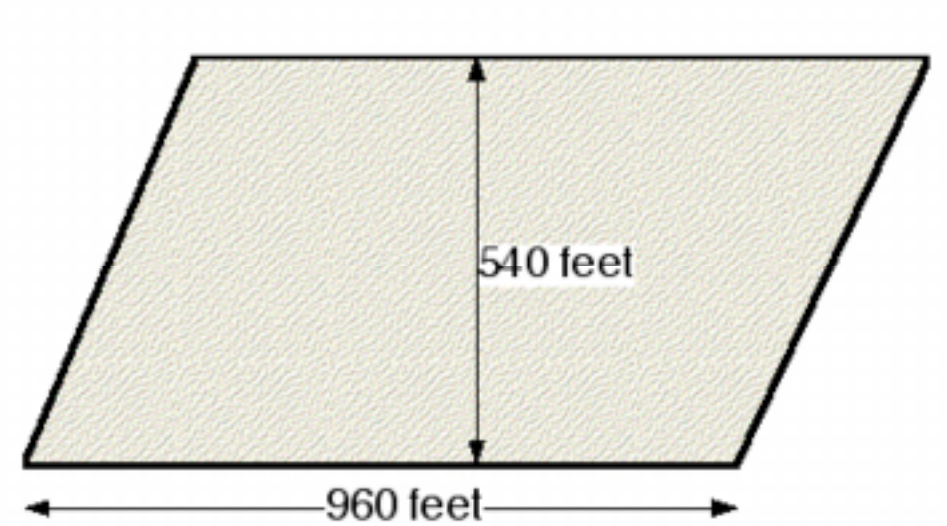
b = 40 yards = (40)(3) feet = 120 feet
h = 40 yards = (20)(3) feet = 60 feet

Now we find the area, in square feet, of the triangular region.
Area = (1/2)bh = (1/2)(120 feet)(60 feet) = 3,600 square feet.

Finally, we multiply by the cost factor of $1.25 per square foot:

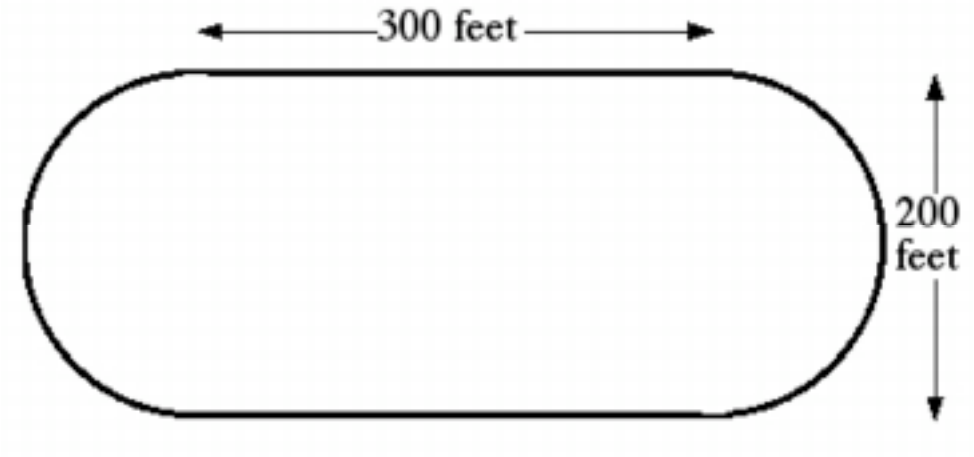
Cost = (3600 sq. ft.)($1.25 per sq. ft.) = $4500
The correct choice is C.

**EXAMPLE 3.8.2**
The diagram below shows the plan for a new parking lot at the Southwestdale Mall. It is estimated that such construction costs $12 per square yard. What will be the total cost for the parking lot?



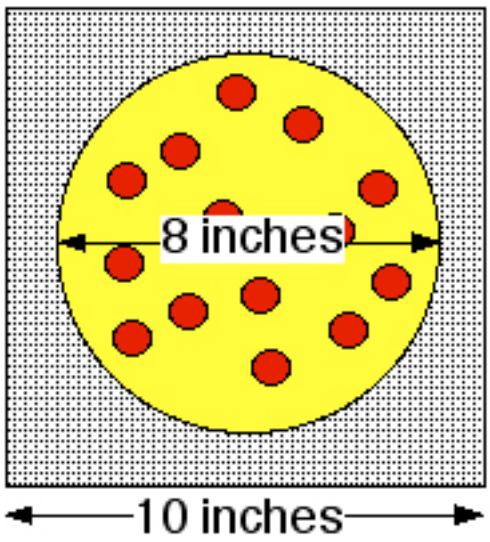

A. $2,073,600
B. $691,200
C. $6,220,800
D. $518,400

EXAMPLE 3.8.3
The area enclosed by the racetrack below will be landscaped. Find the total cost if landscaping costs $2.00 per square yard.




A. $41,258
B. $60,944
C. $20,315
D. $123,774

**EXAMPLE 3.8.4**
The diagram below shows a circular pizza whose diameter is 8 inches, situated in a square box whose side length is 10 inches. How much of the box is "empty?"




**EXAMPLE 3.8.4 solution**
The area of the "empty" part of the box is the area of the shaded region of the figure shown above. Since the shaded region corresponds to the region that remains after a circle has been removed from a square, we make this observation about area:

Area of shaded region = area of square – area of circle.

The area of the square is computed as follows:

Area of rectangle = LW = (10 in.)(10 in.) = 100 sq. in.

To compute the area of the circle, we first observe that since the diameter of the circle is 8 inches, its radius is 4 inches. Now we use the formula for the area of a circle: $A = \pi r^2$

Area of circle = $\pi(4^2) = \pi(16) \approx 50.24$ sq. in.

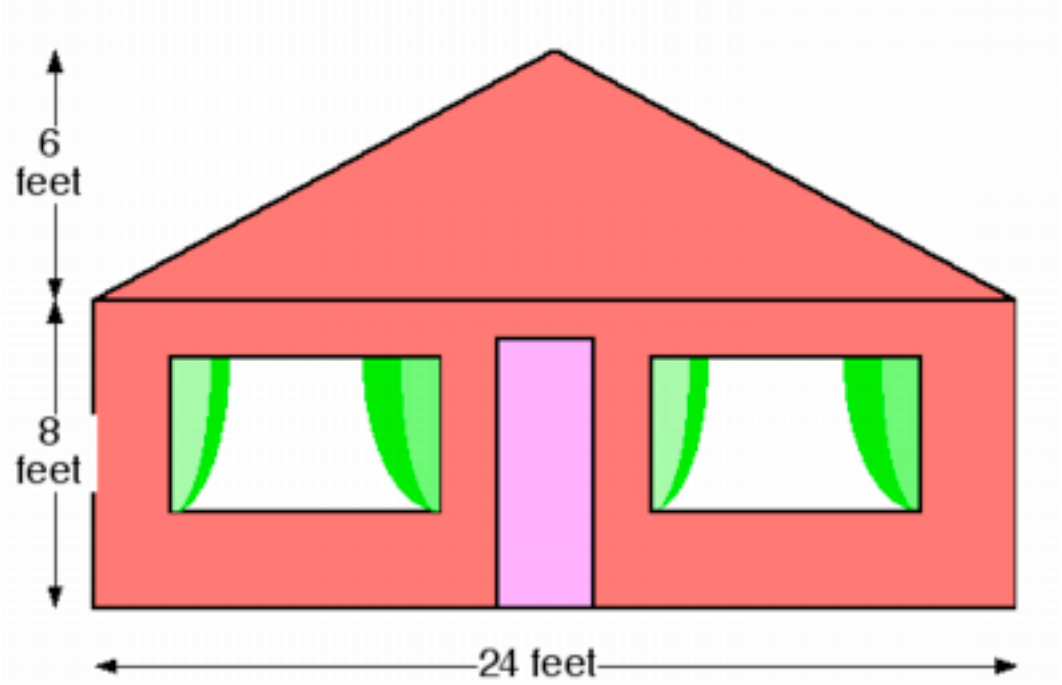
Now we subtract:

Area of shaded region = 100 - 50.24 = 49.76 sq. in.

Comparing the area of the "empty part" of the box (49.76 sq. in.) with the area of the entire box (100 sq. in.), we see that when a 10 inch by 10 inch square box is used to contain an 8-inch-diameter circular pizza, the box is approximately half empty.

**EXAMPLE 3.8.5**

The diagram below shows one exterior wall of a house. The wall has a door opening that measures 3 feet by 7 feet, and two window openings that each measure 6 feet by 4 feet. The wall, but not the openings, will be covered with siding material that costs $1.50 per square foot. Find the total cost of the siding material.




A. $400.50 B. $292.50 C. $499.50 D. $333.00

**EXAMPLE 3.8.6**
Which is larger: a square cake pan that measures 8 inches on each side, or a circular cake pan with a diameter of 9 inches?

**EXAMPLE 3.8.7**
A real-life encounter with GEOMETRY.

An acre is about 40,000 square feet, so you can imagine an acre to be a square area measuring 200 feet on each side. Several years ago, I was interested in hiring a bulldozer operator to clear a square parcel of land that measured 100 feet on each side. During a telephone discussion, one contractor offered the following: "I charge $600 per acre. An acre measures 200 by 200 feet, so a 100 by 100 foot plot would cost $300."

Why did I hang up?

**EXAMPLE 3.8.8**
Suppose that it costs $480 to build a rectangular wooden deck that measures 6 feet by 8 feet. Assuming that the cost of such an object depends upon its size (area), how much would it cost to build a similar deck measuring 18 feet by 24 feet?
A. $640 B. $1440 C. $1,920 D. $4,320

**EXAMPLE 3.8.9**
A circular pizza that is 16 inches in diameter costs $12.00. Assuming that the cost of such a pizza depends upon its size (area), what would be the cost of a pizza that is 8 inches in diameter?
A. $3.00 B. $4.00 C. $6.00 D. $8.00

**EXAMPLE 3.8.10**
The morning after a party, Gomer finds on his living room carpet a circular purple wine stain with a diameter of 1 foot. Homer's Carpet Service charges him $30 to remove the stain. Assuming that the cost of removing such a stain depends upon its size, how much would it cost to remove a stain that 18 inches in diameter?
A. $45.00 B. $67.50 C. $54.00 D. $90.00

**EXAMPLE 3.8.11**
Suppose that it takes 12 hours to decontaminate a circular chemical spill that has a radius of 6 feet. Assuming that the amount of time required to decontaminate such a spill depends upon its size (area), how many hours would it take to decontaminate a similar spill with a radius of 3 feet?
A. 4.5 hours B. 6 hours C. 1.5 hours D. 3 hours

**EXAMPLE 3.8.11 solution**

One method for solving this type of problem is to use a proportion.

To use a proportion, we rely on the following observation:
The number of hours required to decontaminate the smaller spill, in proportion to the area of the smaller spill, should be equal to the number of hours required to decontaminate the larger spill in proportion to its area.

$$\frac{\text{number of hours for smaller spill}}{\text{area of smaller spill}} = \frac{\text{number of hours for larger spill}}{\text{area of larger spill}}$$

$$\frac{x}{\pi(3^2)\text{ sq. ft.}} = \frac{12\text{ hours}}{\pi(6^2)\text{ sq. ft.}}$$

$$\frac{x}{28.274\text{ sq. ft.}} = \frac{12\text{ hours}}{113.097\text{ sq. ft.}}$$

We solve this proportion for x by using a variation of “cross multiplication.”

$$x = \frac{12 \text{ hours} \times 28.274 \text{ sq. ft.}}{113.097 \text{ sq. ft.}}$$

$$x = \frac{339.288 \text{ hours}}{113.097} \approx 2.9999 \text{ hours}$$

**EXAMPLE 3.8.12**
Gomer's bathroom wall measures 8 feet high and 8 feet wide. He is going to cover the wall with square tiles that measure 2 inches by 2 inches. How many tiles are required to cover the wall?

**WORLD WIDE WEB NOTE**
For more practice problems, visit the companion website and try THE GEOMETRIZER.

**PRACTICE EXERCISES**

**1.** A circular pizza pan whose diameter is 18 inches costs $15. Assuming that cost depends upon the size (area) of the pan, what would be the cost of a similar pan whose diameter is 9 inches?
A. $30B. $7.50 C. $6 D. $3.75

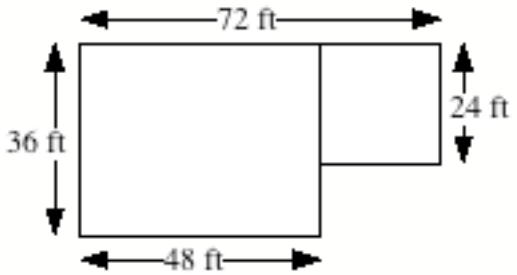
**2.** A carpet-cleaning service estimates that it will cost $40 to remove a circular stain that is 12 inches in diameter. Assuming that the cost of removing a stain depends upon the size (area) of the stain, what would be the cost of removing a similar stain whose diameter is 18 inches?
A. $90B. $60C. $26.67 D. $160

**3.** Aristotle is going to use fabric to cover one of the interior walls of his olive oil warehouse. The wall is 60 feet long and 12 feet tall. The fabric is measured in square yards. How many square yards of fabric will be required to cover the wall.
A. 24 B. 9 C. 72 D. 80

**4.** Euclid has a contract to trim weeds around the grave markers at the local cemetery. He estimates that for this kind of work, it will take two hours to complete the work on one acre. The cemetery is rectangular, measuring 220 feet by 880 feet. Approximately how long will it take for Euclid to complete the job? (Assume that one acre is roughly equal to 40,000 square feet.)
A. 12.5 hours B. 9 hours C. 4.5 hours D. 2.25 hours

**5.** What is the area of a circular region whose radius is 8 inches?
A. 64π inches B. 64π sq. inches C. 64π cu. inches D. none of these

**6.** The diagram below shows the floor plan for a house. If the cost of construction is $80 per square foot, how much will this house cost?




A. $184,320
B. $144,000
C. $322,560
D. $253,440

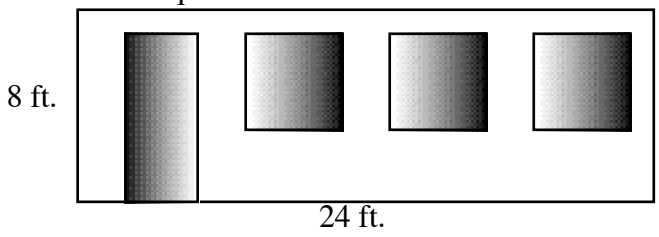
**7.** Suppose that it takes three quarts of paint to cover a rectangular floor that is 12 feet wide and 16 feet long. Assuming that the amount of paint required depends upon the size (area) of the floor, how much paint would be required to cover a floor that is 24 feet wide and 32 feet long?

A. 8 quarts B. 6 quarts C. 12 quarts D. 4 quarts

**8.** Suppose that cleaning up an oil slick from the surface of a lake costs $200,000 if the oil slick is circular in shape with a radius of 2 miles. Assuming that the cost of cleaning up the oil slick depends upon its size (area), what would be the cost of cleaning up a circular oil slick with a radius of 1/2 mile?

A. $100,000 B. $50,000 C. $25,000 D. $12,500

**9.** The diagram below shows one exterior wall of a house. The wall has a door that measures 3 ft. by 7 feet, and three windows which each measure 4 ft. by 4 ft. The wall (but not the doors or windows) will be covered with siding material. How much siding material is required?




A. 192 sq. ft.
B. 155 sq. ft.
C. 123 sq. ft.
D. 261 sq. ft.

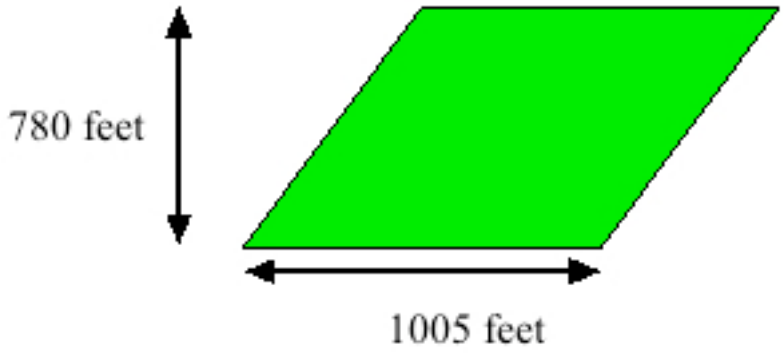
**10.** Referring to the situation in the previous problem: A special window treatment requires a fabric that costs $18.00 per sq. yd. What will be the total cost of window treatment for the three windows shown?

A. $96 B. $288 C. $2592 D. $864

**11.** A rectangular section of wall measuring 14 feet by 6 feet will be covered with square tiles measuring 4 inches by 4 inches. Approximately how many tiles are needed to cover the section of wall?

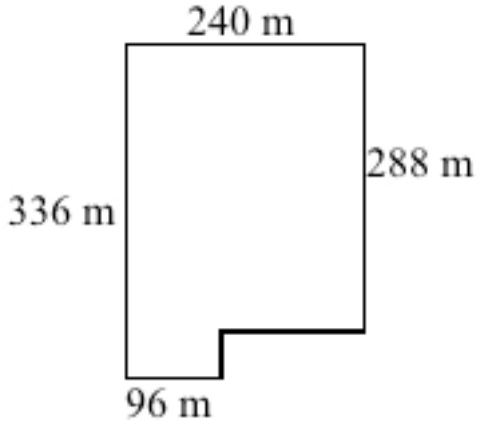
A. 1323 B. 378 C. 1008 D. 756

**12.** The figure below shows the plan for the a new county park. How many acres will the park occupy? (Use the estimate: 1 acre = 40,000 square feet.)




A. 6.53 acres B. 391950 acres
C. 176.38 acres D. 19.60 acres

**13.** The figure below shows the parcel of land on which Aristotle the rancher confines his giraffes. His rule of thumb dictates that each giraffe requires 500 square meters of space. Approximately how many giraffes can the parcel accommodate?




A. 2 B. 4608 C. 161 D. 147

**ANSWERS TO LINKED EXAMPLES**

**EXAMPLE 3.8.2** B
**EXAMPLE 3.8.3** C
**EXAMPLE 3.8.7** He was going to charge twice as much as he should have according to his stated rate of $600 per acre.
**EXAMPLE 3.8.8** D
**EXAMPLE 3.8.9** A
**EXAMPLE 3.8.10** B
**EXAMPLE 3.8.12** 2304 tiles

**ANSWERS TO PRACTICE PROBLEMS**

| | | | | | |
|---|---|---|---|---|---|
| **1.** D | **2.** A | **3.** D | **4.** B | **5.** B | **6.** A |
| **7.** C | **8.** D | **9.** C | **10.** A | **11.** D | **12.** D |
| **13.** D | | | | | |