Use mathematical induction to prove: $5^{2n-1} + 1$ is divisible by 6, for \( n = 1, 2, 3, \ldots \)

Let \( P(n) \) be the proposition “$5^{2n-1} + 1$ is divisible by 6.”

**Basis step:**
Prove \( P(1) \).
Proof: $5^{2(1)-1} + 1$ is a multiple of 6 because $5^{2(1)-1} + 1 = 5+1 = 6$.

**Inductive step**
Prove \( P(k) \rightarrow P(k+1) \)

(That is, show that if $5^{2k-1} + 1$ is divisible by 6, then $5^{2(k+1)-1} + 1$ is divisible by 6.

Proof:
Assume that $5^{2k-1} + 1$ is divisible by 6.
Then there is an integer \( a \), such that $5^{2k-1} + 1 = 6a$.

Then
\[
5^{2(k+1)-1} + 1 \\
= 5^{2k+2-1} + 1 \\
= 5^{2+2k-1} + 1 \\
= 5^25^{2k-1} + 1 \text{ (property of exponents)} \\
= 25(5^{2k-1}) + 1 \\
= 25(5^{2k-1} + 1) + 1 - 25 \text{ (adding 25 inside parentheses, subtracting 25 outside)} \\
= 25(5^{2k-1} + 1) - 24 \\
= 25(6a) - 24 \text{ (inductive hypothesis)} \\
= 25(6a) - 4 \cdot 6 \\
= 6(25a - 4) \text{ (factoring)} \\
= 6b \text{ where } b = 25a - 4