Use mathematical induction to prove the following:

\[ 3n + 4 \leq n^2 \text{ for } n = 4, 5, 6, \ldots \]

Proof

Let \( P(n) \) be the predicate \( 3n + 4 \leq n^2 \).

Basis
We need to prove \( P(4) \).
That is, we need to prove \( 3(4) + 4 \leq 4^2 \).

Proof of basis: \( 3(4) + 4 = 16 \) and \( 4^2 = 16 \) so \( 3(4) + 4 \leq 4^2 \).

Inductive
We need to prove \( P(k) \rightarrow P(k+1) \).
That is, we need to prove \( 3k + 4 \leq k^2 \rightarrow 3(k+1) + 4 \leq (k+1)^2 \)

Prove: Assume \( 3k + 4 \leq k^2 \) for arbitrary \( k \geq 4 \).

Then
\[
3(k+1) + 4 \\
= 3k + 3 + 4 \\
= 3k + 4 + 3 \\
\leq k^2 + 3 \quad \text{(inductive hypothesis)} \\
< k^2 + 2k + 1 \quad \text{(because if } k \geq 4, \text{ then } 2k + 1 \geq 9 > 3) \\
= (k+1)^2
\]