Friday, February 1

Today we will finish Course Notes 2.3, begin 3.1.

Tuesday in recitation you have Test 1, covering 2.1, 2.2, 2.3, 1.1, and the beginning of 3.1.

You also have an online quiz, covering some Test 1 topics, due by 11:59 p.m., Tuesday.
**DeMorgan’s Laws for Nested Quantifiers**

If there is more than one quantifier, then the negation operator should be passed from left to right across one quantifier at a time, using the appropriate DeMorgan Law at each step.

**EXAMPLE**

Use DeMorgan’s Laws to negate:

\[ \exists x \forall y [Q(x) \land R(x, y)] \]
Let $Q(x): x$ is a rational number; $I(x): x$ is an irrational number. The universe for $x$ is the set of real number.

Evaluate each of these quantified propositions:

1. $\forall x [Q(x) \lor I(x)]$

2. $\forall x Q(x) \lor \forall x I(x)$

Which propositions are true?

A. Prop. 1 only is true
B. Prop. 2 only is true
C. Prop. 1 and Prop. 2 are both true
D. Both are false
The result of the previous exercise proves the following fact (from the end of Chapter 2 Section 3) about distributing quantifiers.

*For any propositions* \( P(x), Q(x), \)

\[
\forall x[P(x) \lor Q(x)] \quad \quad \quad \quad \forall xP(x) \lor \forall xQ(x)
\]

That is, the *universal* quantifier does/does not distribute over a *disjunction*.
Referring to the previous example, note that

$$\exists x [Q(x) \land I(x)]$$ is ___________.  $$\exists x Q(x) \land \exists x I(x)$$ is ___________.

This proves:

*For any predicates* $P(x)$, $Q(x)$,

$$\exists x [P(x) \land Q(x)]$$ _________  $$\exists x P(x) \land \exists x Q(x)$$

That is, the *existential* quantifier does/does not distribute over a *conjunction*.

Finally, two other facts that cannot be proven with just a single example:

$$\exists x [P(x) \lor Q(x)]$$ _________  $$\exists x P(x) \lor \exists x Q(x)$$

$$\forall x [P(x) \land Q(x)]$$ _________  $$\forall x P(x) \land \forall x Q(x)$$
Course Notes 3.1 Arguments and Inference

Argument

An argument consists of one or more propositions, called the hypotheses or premises, followed by a proposition called the conclusion.

EXAMPLE
I don’t own a badger.
If today is Thursday, then I own a badger.
Therefore, today isn’t Thursday.

Valid/Invalid arguments (working definitions)

We say that an argument is valid if it is impossible for the conclusion to be false, when every premise is assumed to be true. “The truth of the premises forces the truth of the conclusion.”

On the other hand, if it is possible for the conclusion to be false while every premise is assumed to be true, we say that the argument is invalid.
The working definition of invalid argument gives rise to a technique for analyzing certain kinds of arguments:

**Using a truth table to test the validity of an argument**

One way to decide if an argument is valid is by using a truth table:

1. Symbolize the premises and the conclusion of the argument.

2. Make a truth table that has columns for each of the premises $p_1, p_2, \ldots p_n$ and a column for the conclusion $c$.

3. If there is at least one row where every premise column is true while the conclusion column is false, then the argument is invalid.

4. If there is no row where every premise column is true while the conclusion column is false, then the argument is valid.
Example

*Use a truth table to test the validity of the following argument.*

If I enter the poodle den, then I will carry my electric poodle prod or my can of mace. I am carrying my electric poodle prod but not my can of mace. Therefore, I will enter the poodle den.
Consider Argument 1 and Argument 2.

<table>
<thead>
<tr>
<th><strong>Argument 1</strong></th>
<th><strong>Argument 2</strong></th>
<th></th>
</tr>
</thead>
</table>
| *If that animal is a wolverine, then it is bitey.*  
That animal is bitey.  
Therefore, that animal is a wolverine. | *If that animal is a wolverine, then it is bitey.*  
That animal is a wolverine.  
Therefore, that animal is bitey. | A. Only Argument 1 is valid.  
B. Only Argument 2 is valid.  
C. Both are valid.  
D. Both are invalid. |